

DECAY OF THE HIGGS BOSON INTO A Z BOSON - PHOTON PAIR IN THE STANDARD MODEL AND BEYOND

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Abstract

In this work we study the computation of the branching ratio of the Higgs boson decaying into a Z boson-photon pair at leading order. The calculation was performed in detail using the unitary gauge, the obtained results were contrasted with the corresponding expressions in alternative gauge choices. The amplitudes of the dominant contributions, coming from diagrams with a W boson in the loop corrections, were constructed using the path integral formulation and verified with the diagrammatic method of Feynman. The corresponding loop integrals were reduced by using the Passarino-Veltman method. The decay width obtained depends on the square of one of the form factors derived in the tensor reduction. Furthermore, the computation was automated using Wolfram Mathematica in conjunction with the packages FeynArts, FeynCalc, and FeynHelpers, yielding the analytical expression for the decay width. Numerical results obtained using Python yielded a partial decay width at leading order (LO) of 8.33 keV and a corresponding branching ratio of 2.25×10^{-3} . These results indicate a discrepancy of 1.15×10^{-3} when compared to the experimental branching ratio, $(3.4 \pm 1.1) \times 10^{-3}$, for a Higgs boson mass of 125 GeV.

Keywords: Decay Width, Higgs Boson, Photon, Z-Boson.

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1. Introduction

The study of the Higgs boson properties is a key focus for the agenda in future accelerators, such as the ILC, FCC and HL-LHC, since they aim to uncover new physics that could address problems without solutions in the SM. There are well motivated new physics models that include supersymmetry (SUSY) as a fundamental feature, with a region of parameters suitable to explain current experimental deviations from SM predictions, such as the W boson mass [1–4] and the muon anomalous magnetic moment [5, 6], besides offering natural dark matter candidates.

Physical observables associated with various Higgs boson decay modes have been measured, reflecting its short lifetime as an unstable particle. In particular, the analysis of the di-photon decay channel led to the discovery of the Higgs boson by the ATLAS and CMS collaborations in 2012 [7, 8]. Notably, the decay of the Higgs boson into a photon and a Z boson has been observed at the LHC, albeit with a low branching ratio [9]. Although this process is rare, it is a valuable observable for potential searches of new physics as it is a process that can be computed starting at the one-loop level and, therefore, is sensitive to Beyond Standard Model (BSM) contributions that can be tested against experimental results.

In the SM, the decay width of the Higgs boson into a Z boson-photon pair has been calculated up to next-to-leading order (NLO). The full contribution at leading order (LO) is very well known in different gauges [10–12], while at NLO, the computation includes both QCD [13–15] and electroweak corrections [16, 17]. Outstandingly, the QCD correction, amounting to 0.3 % of the LO result, to the top quark loop is smaller than the NLO electroweak correction, which accounts for 7 % of the LO prediction. At this level of precision, the theoretical prediction shows a significant discrepancy with the experimental data, with the signal yield being 2.2 ± 0.7 times the SM prediction, highlighting the need for next-to-next-to-leading order (NNLO) calculations to assess whether the SM contribution could explain the differences with the expected results in the RUN3 at the HL-LHC. This decay process has also been explored in other BSM scenarios [10, 18, 19], which could provide a viable explanation for the discrepancy between SM predictions and experimental

results. Similar computations have been performed within supersymmetric models, such as the Minimal Supersymmetric Standard Model (MSSM) [10], the Next-to-Minimal Supersymmetry Model (NMSSM) [20], low-energy SUSY models like the Constrained Minimal Supersymmetric Standard Model (CMSSM) and the Nearly Minimal Supersymmetric Standard Model (nMSSM) [21], as well as the $(B - L)$ Supersymmetric Standard Model (BLSSM) [22].

To study the decay width of the Higgs boson into a Z boson and a photon at LO, this work is structured into two chapters. Chapter 2 presents the electroweak Standard Model using the functional formalism, discusses the relevant Lagrangian for the W boson loop contribution, introduces the Feynman rules related to the decay process, and examines the formula for the unpolarized decay width in detail. The current state of experimental and theoretical results on Higgs boson decays is also reviewed. Chapter 3 focuses on the analytical calculation of the dominant W boson loop contribution to the decay width of the Higgs boson into a Z boson and a photon. This begins with obtaining the three-point Green's function using functional methods, followed by constructing the amplitude through the LSZ reduction formula. The loop integral is then solved using Passarino-Veltman tensor reduction. Once the amplitude is computed, the unpolarized decay width is determined. Additionally, a Mathematica package is introduced to automate the calculation of the decay width, enabling analysis of its behavior across different scales. The results are compared with those from other studies. Finally, in Chapter 4, we present our conclusions. Appendices are included to detail specific procedures. In Appendix A, the notation and conventions used for the computations are outlined. In Appendix B, additional computations are provided, including: the ghost propagator, with a discussion on how it vanishes in the unitary gauge; the derivation of the Green function using functional methods; the application of the LSZ reduction formula to obtain the amplitude; the derivation of the contributing form factors via the Ward-Takahashi identity; the reduced formula for the partial decay width; the tensor reduction procedure; and a discussion of the Passarino-Veltman functions required for the contributing form factor.

2. Electroweak Standard Model

This chapter provides a detailed overview of the formalism governing electroweak interactions, including the role of the Higgs boson, and introduces the concept of path integrals. It also explores the relevant Lagrangians for the $H \rightarrow Z\gamma$ decay process, which are essential for calculating the associated amplitude. The chapter begins by discussing the electroweak Lagrangian derived from Yang-Mills theory. It then covers the Feynman rules pertinent to the decay process at LO, followed by the expression used to compute the decay width. Finally, experimental results related to this process will be presented and discussed.

2.1 Electroweak Model Lagrangian

The decay process developed in this work requires to consider the electroweak SM, a non-Abelian gauge theory which includes the Yang-Mills Lagrangian for the SU(2) internal symmetry to describe interactions, but also the Dirac Lagrangian for matter fields. To incorporate photons in this model, the SU(2) symmetry group of weak interactions is extended by an Abelian group, U(1), leading to the symmetry group $SU(2)_L \otimes U(1)_Y$, where L stands for left-handed (since fermions violate parity in weak nuclear processes) while Y refers to the hypercharge quantum number associated with each field.

Explicitly, the Yang-Mills term is:

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}W^{i\mu\nu}W_{\mu\nu}^i, \quad (2.1)$$

where $B^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$ is the strength field tensor of the U(1) gauge field B^μ and $W_i^{\mu\nu} = \partial^\mu W_i^\nu - \partial^\nu W_i^\mu + g\varepsilon_{ijk}W_j^\mu W_k^\nu$ is the strength field tensor for the three SU(2) gauge bosons $W_{i=1,2,3}^\mu$. The gauge fields B^μ, W_3^μ can be related to the physical fields of the photon, A^μ , and Z-boson, Z^μ , through the Weinberg rotation,

$$\begin{pmatrix} B^\mu \\ W_3^\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix}, \quad (2.2)$$

where we have used the short-hand notation: $\sin\theta_W = s_W$, $\cos\theta_W = c_W$. The other W_1^μ and W_2^μ fields are mixed to define the charged massive vector bosons $W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2)$.

Since weak nuclear force has a small finite range of interaction, the vector bosons $W^\pm$ and Z are massive and, therefore, it is essential to introduce the Higgs mechanism to generate their masses, but also to save unitarity and renormalizability of the theory [23–26]. In order to obtain such masses, we need to consider the couplings between Higgs and gauge boson fields contained in the Higgs Lagrangian:

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi); \quad \text{with} \quad V(\phi) = m^2 |\phi|^2 + \lambda |\phi|^4, \quad (2.3)$$

where

$$D^\mu = \partial^\mu - igW_i^\mu T_i - ig' \frac{Y}{2} B^\mu, \quad (2.4)$$

is the covariant derivative that ensures gauge invariance of the Lagrangian. Besides, Y is the hypercharge and T_i are the SU(2) generators in the adjoint representation. The coupling constants, g and g' , satisfies the relation $e = gs_W = g'c_W$, where e is the positron electric charge. Finally, ϕ is a doublet of scalar bosons defined, after spontaneous symmetry breaking, as

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} G^\pm(x) \\ [v + H(x) + iG^0(x)] \end{pmatrix}, \quad G^+(x) = [G^-(x)]^\dagger, \quad (2.5)$$

where $G^i(x)$ are the would-be Goldstone bosons and $H(x)$ represents the physical Higgs field. By replacing the Higgs doublet in the expression (2.3) and after performing the Weinberg rotation, the Lagrangian provides the Higgs-gauge boson couplings and the masses of the gauge bosons [27–29]:

$$\mathcal{L}_{\text{Higgs}} \approx \frac{1}{2} \partial_\mu H \partial^\mu H - \frac{1}{2} M_H^2 H^2 + \left(M_W^2 W^{\mu+} W_\mu^- + \frac{1}{2} M_Z^2 Z^\mu Z_\mu \right) \left(1 + \frac{H}{v} \right)^2, \quad (2.6)$$

where $M_Z = \frac{e^2}{4} \left(\frac{1}{s_W^2 c_W^2} \right) v^2$, $M_W = \frac{1}{4} \frac{e^2}{s_W^2} v^2$, and $M_H = \sqrt{2\lambda} v^2$. Here, λ is the coupling constant for the quartic self-interaction of the Higgs field, and v is the vacuum expectation value (vev), while the photon remains massless, as expected. Additionally, $M_Z^2 = \frac{M_W^2}{c_W^2}$.

In order to select a single solution from infinitely many equivalent states related through the SU(2) symmetry transformation, we need to fix the gauge using a gauge

function:

$$\mathcal{L}_{GF} = -\frac{1}{2\xi} (\partial^\mu W_\mu^i)^2 - \frac{1}{2\xi} (\partial^\mu B_\mu)^2. \quad (2.7)$$

Gauge-fixing terms are included to address theoretical redundancies in the path integral quantization for gauge bosons and to derive the gauge propagators. Thereby, we have the useful terms in order to obtain the scattering amplitude for the $H \rightarrow Z\gamma$ process.

We adopt the unitary gauge ($\xi \rightarrow \infty$) to avoid Goldstone bosons. Additionally, in this gauge, ghost contributions vanish because their mass depends on ξ , making them static in this context. A detailed computation of the ghost Lagrangian and propagator via functional methods [30] is provided in Appendix (B.1),

$$\begin{aligned} \mathcal{L}_{\text{free}} = & \mathcal{L}_{GF} + \frac{1}{2} \partial^\mu H \partial_\mu H - \frac{1}{2} M_H^2 H^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \\ & - \frac{1}{4} W^{+\mu\nu} W_{\mu\nu}^- - \frac{1}{4} Z^{\mu\nu} Z_{\mu\nu} - \frac{1}{2} M_Z^2 Z^\mu Z_\mu - M_W^2 W^{+\mu} W_\mu^-, \end{aligned} \quad (2.8)$$

where the required gauge-fixing Lagrangian is

$$\mathcal{L}_{GF} = -\frac{1}{2\xi} (\partial \cdot A)^2 - \frac{1}{2\xi} (\partial \cdot Z)^2 - \frac{1}{\xi} (\partial \cdot C^+) (\partial \cdot C^-). \quad (2.9)$$

With the gauge function $C^\pm$ defined in (B.2), this leads to ghost contributions.

In order to build the amplitude for the dominant contribution coming from the vertex loop with a virtual W boson, the required Lagrangian terms for the triangle loop are:

$$\mathcal{L}_{HWW} = \frac{eM_W}{s_W} H W_\mu^+ W_\mu^-, \quad (2.10)$$

$$\mathcal{L}_{AWW} = i e (W_{\mu\nu}^+ W_\nu^\mu A^\nu - W_{\mu\nu}^- W_\nu^\mu A^\nu + F_{\mu\nu} W_\nu^\mu W_\nu^\nu), \quad (2.11)$$

$$\mathcal{L}_{ZWW} = -i \frac{e}{s_W} (W_{\mu\nu}^+ W_\nu^\mu Z^\nu - W_{\mu\nu}^- W_\nu^\mu Z^\nu + Z_{\mu\nu} W_\nu^\mu W_\nu^\nu), \quad (2.12)$$

which contain trilinear couplings written in terms of physical observables [31]. In the $H \rightarrow Z\gamma$, the loop diagram with the quartic interaction vertex $W^- W^+ Z A$ can be required to remove UV divergences, the required interaction term is:

$$\mathcal{L}_{WWZA} = \frac{e^2 c_W}{s_W} (2g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) W_\mu^+ W_\nu^- A_\rho Z_\sigma. \quad (2.13)$$

Calculating the amplitude requires the corresponding S-matrix element, which

encapsulates the contributions from the interaction terms in the above Lagrangians. The S-matrix acts as a fundamental link between the theoretical framework defined by the Lagrangian and the observable quantities in scattering or decay processes.

$$\langle f | S | i \rangle = \delta_{fi} + i (2\pi)^4 \delta^{(4)}(p_f - p_i) \mathcal{M}_{fi}. \quad (2.14)$$

In order to obtain S-matrix elements, we introduce the n-point Green function via functional formalism

$$\langle \Omega | T \{ \Phi_1^i(x_1) \dots \Phi_n^j(x_n) \} | \Omega \rangle = \frac{\delta^{(n)}}{\delta J_1(x_1)_i \dots \delta J_n(x_n)_j} \left(\frac{Z[J_1, \dots, J_n]}{Z[0, \dots, 0]} \right)_{J_k=0}, \quad (2.15)$$

with the generating functional

$$Z[J_1, \dots, J_n] = \int \mathcal{D}\Phi_1 \dots \mathcal{D}\Phi_n e^{i \int d^4x (\mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}} - i J_1^i \Phi_1^i - \dots - i J_n^j \Phi_n^j)}. \quad (2.16)$$

Here, n refers to different external fields, and j represents various types of indices, such as Lorentz, Dirac or color indices, depending on the field type. The main aim of our work is to find the on-shell amplitude

$$\mathcal{M}(H \rightarrow Z\gamma) = \varepsilon_{\mu}^{\lambda_2*}(p_2) \varepsilon_{\nu}^{\lambda_3*}(p_3) \mathcal{R}^{\mu\nu}. \quad (2.17)$$

Since the Ward-Takahashi identity is satisfied, $p_3^{\mu} \mathcal{R}_{\mu\nu} = 0$, the longitudinal polarization of the external photon is decoupled. Consequently, the result for the amplitude is gauge-independent. Therefore, we choose the unitary gauge ($\xi \rightarrow \infty$) to eliminate the Goldstone contributions, allowing us to work only with physical fields.

2.2 Feynman Rules

The Feynman rules can be derived via the functional methods explained above. Specifically, the Feynman rules relevant to the $H \rightarrow Z\gamma$ process are provided by the free and interacting Lagrangian terms (2.8), (2.11), (2.10), (2.12) and (2.13), and we obtain the following rules:

$$\text{Higgs boson propagator:} \quad \text{---} \text{---} \frac{p}{p^2 - M_H^2} \text{---} \text{---} = \frac{i}{p^2 - M_H^2}. \quad (2.18)$$

Massive gauge boson propagator: $\mu \text{---} \text{wavy line with } p \text{---} \nu = \tilde{\Delta}_V^{\mu\nu}(p).$ (2.19)

Photon propagator: $\mu \text{---} \text{wavy line with } p \text{---} \nu = \tilde{\Delta}_\gamma^{\mu\nu}(p),$ (2.20)

where the expressions for vector boson propagators in a general gauge are:

$$\tilde{\Delta}_V^{\mu\nu}(p) = \frac{i}{p^2 - M_V^2} \left(-g^{\mu\nu} + (1 - \xi) \frac{p^\mu p^\nu}{p^2 - \xi M_V^2} \right), \quad (2.21)$$

$$\tilde{\Delta}_\gamma^{\mu\nu}(p) = \frac{i}{p^2} \left(-g^{\mu\nu} + (1 - \xi) \frac{p^\mu p^\nu}{p^2} \right). \quad (2.22)$$

On the other hand, the Feynman rules for the required couplings are:

Trilinear gauge coupling: $V_\mu^1, p_1 \text{---} V_\nu^2, p_2 \text{---} V_\rho^3, p_3 = ieC\mathcal{V}_{\mu\nu\rho}(p_1, p_2, p_3).$ (2.23)

Higgs - W boson coupling: $H \text{---} W^+ \text{---} W^- \text{---} \nu = \frac{ieM_W}{s_W} g_{\mu\nu}.$ (2.24)

Quartic gauge coupling: $\mu \text{---} W^+ \text{---} Z \text{---} \sigma \text{---} \nu \text{---} W^- \text{---} \gamma \text{---} \rho = ie^2 \frac{c_W}{s_W} \mathcal{B}_{\mu\nu\sigma\rho}.$ (2.25)

Additionally, we defined the antisymmetric tensor [32, 33],

$$\mathcal{V}_{\mu\nu\rho}(p_1, p_2, p_3) = g_{\mu\nu}(p_1 - p_2)_\rho + g_{\nu\rho}(p_2 - p_3)_\mu + g_{\rho\mu}(p_3 - p_1)_\nu, \quad (2.26)$$

and the tensor for the quartic vertex,

$$\mathcal{B}_{\mu\nu\sigma\rho} = 2g^{\mu\nu}g^{\sigma\rho} - g^{\nu\rho}g^{\mu\sigma} - g^{\rho\mu}g^{\nu\sigma}. \quad (2.27)$$

In the case of the trilinear gauge boson coupling, $C = 1$ for $V^1 = A$, $V^2 = W^+$, $V^3 = W^-$, and $C = -\frac{c_W}{s_W}$ for $V^1 = Z$, $V^2 = W^+$, $V^3 = W^-$. These Feynman rules align with those in [34], from which FeynArts [35] adopts its convention for generating amplitudes.

2.3 Decay Width

The partial decay width can be computed by means of the formula constructed for a resonant stable state decaying into a final set β of particles,

$$d\Gamma(1 \rightarrow \beta) = \frac{1}{2E_1} (2\pi)^4 \delta^{(4)} \left(p_1^\mu - \sum p_\beta^\mu \right) |\overline{\mathcal{M}}(1 \rightarrow \beta)|^2 \prod_f^N \frac{1}{(2\pi)^3} \frac{d^3 p_f}{2E_f}, \quad (2.28)$$

where $\overline{\mathcal{M}}$ is the unpolarized amplitude for the process, in this case, the sum over the photon and Z boson polarizations must be performed, and there is no averaging over the initial state because the Higgs boson is spinless. Although the Higgs boson is an unstable particle, its total decay width, $\Gamma = 3.7$ MeV [36], is much smaller than the Higgs boson mass, $M_H \approx 125$ GeV [36, 37]. Therefore, the conditions for the validity of this formula are satisfied. However, there is currently no established prescription for computing unstable particles' decay width that does not satisfy this limit.

It is convenient to work in the center-of-mass frame of the Higgs boson, where the decay width can be expressed as

$$\Gamma(H \rightarrow Z\gamma) = \frac{1}{16\pi} \frac{1}{M_H} (M_H^2 - M_Z^2) \frac{1}{M_H^2} |\overline{\mathcal{M}}|^2. \quad (2.29)$$

The derivation of the formula above is shown in Appendix (B.5).

The ATLAS and CMS collaborations have observed that the Higgs boson can manifest in various decay channels, as shown in Figure (2.1).

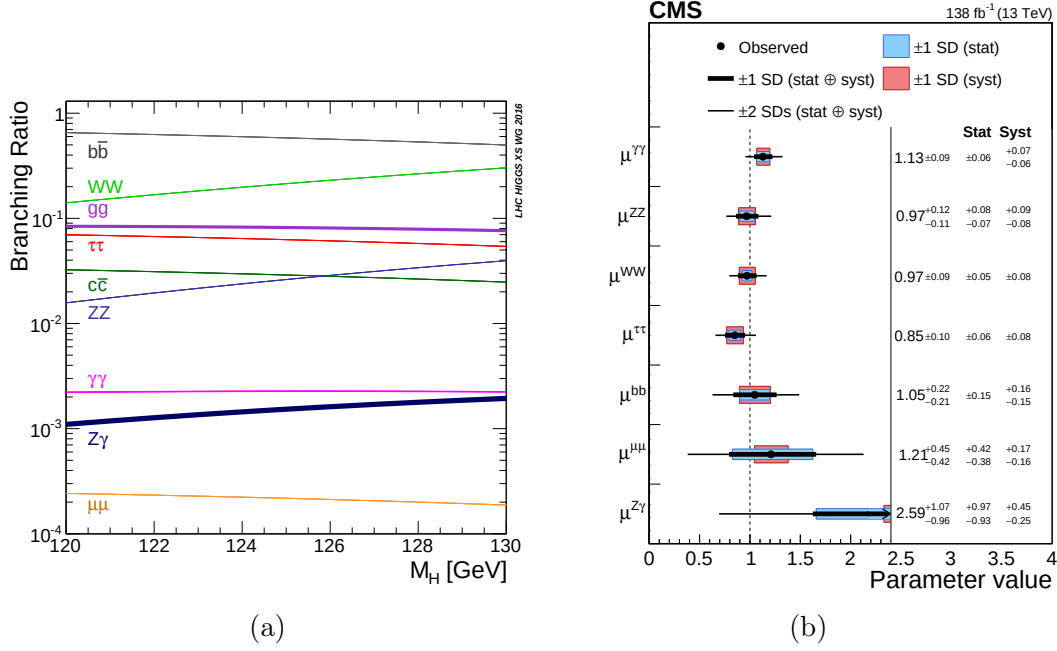


Figure 2.1: (a) Branching ratios for measured decay modes of the Higgs boson [38]. (b) Channel signal strengths (μ) reported by the CMS collaboration. The dashed vertical line centered at 1 represents the signal strength predicted by the SM [39].

It is evident that the $H \rightarrow Z\gamma$ decay is a low-probability process. At LHC, this process has been being searched [40], and recently, the ATLAS and CMS collaborations measured the branching ratio of $H \rightarrow Z\gamma$ to be [9]

$$\text{BR}^{\text{exp}}(H \rightarrow Z\gamma) = (3.4 \pm 1.1) \times 10^{-3}. \quad (2.30)$$

However, since there is no tree-level interaction for this process, at least one loop is required due to the absence of direct interactions between massless particles like photons and the Higgs boson and between neutral particles like the Z boson and photons. This loop allows for virtual corrections and contributions provided by the SM and beyond, making this decay sensitive to new physics. Furthermore, experimental signals for the $H \rightarrow Z\gamma$ decay process have been observed at CERN, as shown in Figure (2.1b). These signals deviate significantly from SM predictions, with a statistical significance of 3.4σ .

The measured signal is 2.2 ± 0.7 times the SM prediction. This opens up the possibility of new physics that could explain the discrepancy with the SM.

3. Higgs Decay into a Photon-Z Boson Pair

In this chapter, we focus on computing of the decay width for the process $H \rightarrow Z\gamma$. First, we construct the amplitude using the functional formalism and the LSZ reduction formula, then we solve the resulting loop integrals via Passarino-Veltman tensor reduction and we include the scattering amplitude in the master formula of the decay width. We also show the systematization of the computation and we compare the numerical results with LHC experimental data.

3.1 Leading Contribution

There are several possible contributions to the decay width of $H \rightarrow Z\gamma$. At the one-loop level, the dominant contribution comes from the diagram with a triangular loop due to its three cubic vertices, which can provide a dominant contribution for large coupling constants. However, the bubble diagram is also necessary because the combined contributions from the bubble and triangle loops ensure a finite result [33]. In the SM, this loop contribution is mediated through fermion and W-boson virtual particles. For fermion loops, as shown in the diagram (a) in Figure (3.1), the top quark provides the only significant contribution because the amplitude is proportional to the fermion mass [13, 15, 41, 42]. In the W boson loop, there is an additional contribution from the two possible orientations of the charge flow within the loop, depicted in Figure 3.1 (b). In this case the electroweak couplings are stronger than the fermion-gauge boson couplings. Thus, numerical results for the LO decay width are significantly larger for the W boson than for the fermionic case [14]. Thereby, we focus on the W-boson contribution, because it is the dominant one.

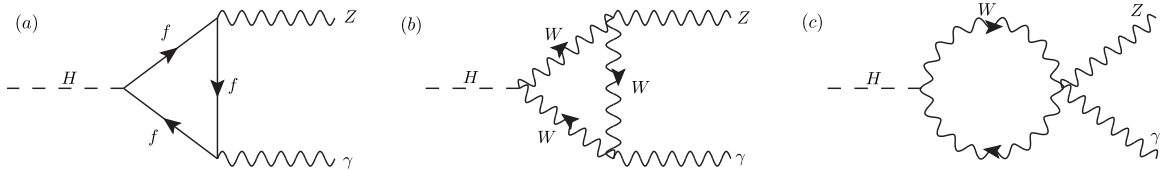


Figure 3.1: Feynman diagrams for fermionic and W boson loop contributions to $H \rightarrow Z\gamma$.

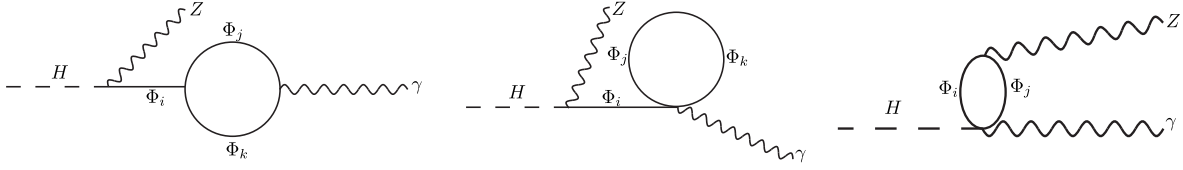


Figure 3.2: Reducible (non-1PI) Feynman diagrams can contain ghosts, W bosons, and Goldstone bosons mediating the loops in the R_ξ gauge. Here, Φ_i represents a field mediating the loop, which can be a W boson, ghost, or Goldstone boson.

The one-loop contribution is finite; forming a counterterm involving the Higgs boson field, the strength tensors for the Z boson, and photon fields is impossible. As was shown in [43], this would imply a Lagrangian term with a mass dimension greater than four, spoiling renormalizability.

There are no contributions from Goldstone bosons and ghost fields in the unitary gauge, where their propagators vanish due to their mass dependence on ξ , as shown in the unphysical part of the gauge boson propagator in (2.21). Additionally, the contributions from ghosts and Goldstone bosons in other gauge choices are negligible compared to the W boson loop contribution [44].

In the general R_ξ gauge, all ghost and Goldstone boson contributions are required. In order to remove UV singularities, non-1PI diagrams are necessary, introducing quantum corrections to external lines through self-energies and tadpoles, as shown in Figure 3.2. Moreover, the absorptive part of the amplitude for these diagrams implies a Cutkosky cut in the loop, resulting in a diagram with two on-shell external lines and the on-shell photon. Since there is only one photon, this configuration violates momentum conservation and is therefore kinematically forbidden [32], resulting in a zero numerical contribution [44]. In the unitary gauge, the sum of the bubble and triangle loops is already finite. Moreover, by considering only 1PI diagrams with all ghost and Goldstone contributions, it is possible to eliminate the pole using the Yennie gauge ($\xi = -3$) [45]. This result was verified using FeynCalc [46, 47],

$$\Delta_\varepsilon = \frac{-i\pi^2 e^3 M_W (\xi + 3)}{2c_W s_W^2} \frac{1}{\varepsilon}. \quad (3.1)$$

3.2 Constructing the Amplitude

To compute the partial decay width, we first need to calculate the unpolarized amplitude and then obtain the squared modulus of the unpolarized amplitude.

3.2.1 Constructing the Green Function

In order to show the cancellations and procedure to obtain the contribution for the triangle diagram for the $H \rightarrow Z\gamma$ process, first, the three-point Green function is constructed,

$$\langle \Omega | T \{ H(x_1) Z_\mu(x_2) A_\nu(x_3) \} | \Omega \rangle = \frac{\delta^{(4)}}{\delta j(x_1) \delta J_0(x_2)^\mu \delta J(x_3)^\nu} \left(\frac{Z[j, J]}{Z[0]} \right)_{J=j=0}, \quad (3.2)$$

where the generating functional reads

$$Z[J, j] = \int \mathcal{D}W^+ \mathcal{D}W^- \mathcal{D}Z \mathcal{D}A \mathcal{D}H e^{i \int d^4x (\mathcal{L}_{\text{int}} + \mathcal{L}_{\text{free}} - i j H - i J^\rho Z_\rho - i J^\sigma A_\sigma - i J_+^\mu W_\mu^- - i W_\mu^+ J_-^\mu)}, \quad (3.3)$$

and the generating functional can be evaluated using the stationary solution via the principle of least action, resulting in an expression where the argument includes the inner product. In this case, the inner product is given by

$(J_V, \Delta_V J_V) = \int d^4x \, d^4y \, J_V^\mu(x) \Delta_V(x-y)_{\mu\nu} J_V^\nu(y)$. Expanding the generating functional as shown in Appendix (B.2), we obtain the first-order perturbative contribution to the Green function.

3.2.2 LSZ Reduction Formula

We have an off-shell object, namely the correlation function, but we require the on-shell amplitude. Therefore, we must to amputate the propagators corresponding to the initial and final states, specifically the Higgs boson, photon, and Z boson propagators. Consequently, the LSZ reduction formula is necessary to obtain the first perturbative

order of the S -matrix element for the decay.

$$\begin{aligned} \langle f | S^{(1)} | i \rangle &= i^{1+2} \int d^4 x_1 e^{-ip_1 x_1} (\partial_1^2 + M_H^2) \int d^4 x_2 \varepsilon_{\lambda_2}^{\mu*}(p_2) e^{+ip_2 x_2} (\partial_2^2 + M_Z^2) \\ &\times \int d^4 x_3 \varepsilon_{\lambda_3}^{\nu*}(p_3) e^{+ip_3 x_3} (\partial_3^2) \langle \Omega | T \{ H(x_1) Z_\mu(x_2) A_\nu(x_3) \} | \Omega \rangle, \end{aligned} \quad (3.4)$$

and recall the construction of the S-matrix element (2.14). Where p_2, p_3 are the four-momentums of the Z-boson and photon, respectively. The final polarized contribution of the triangle loop of the case (b) in Figure (3.1) can be written in the following form,

$$\begin{aligned} i\mathcal{M}^{(b)}(H \rightarrow Z\gamma) &= -2 \frac{e^3 M_W c_W}{s_W^2} \varepsilon_{\lambda_2}^{\mu*}(p_2) \varepsilon_{\lambda_3}^{\nu*}(p_3) \int \frac{d^4 \ell}{(2\pi)^4} \frac{g_{\rho\sigma}}{D_0 D_1 D_2} \left(g^{\rho\alpha} - \frac{\ell^\rho \ell^\alpha}{M_W^2} \right) \\ &\times \mathcal{V}_{\mu\alpha\tau}(-p_2, \ell, -\ell_2) \left(g^{\tau\varepsilon} - \frac{\ell_2^\tau \ell_2^\varepsilon}{M_W^2} \right) \mathcal{V}_{\nu\varepsilon\beta}(-p_3, \ell_2, -\ell_3) \left(g^{\beta\sigma} - \frac{\ell_3^\beta \ell_3^\sigma}{M_W^2} \right), \end{aligned} \quad (3.5)$$

where we have that $D_0 = \ell^2 - M_W^2$; $D_1 = (\ell - p_2)^2 - M_W^2$; $D_2 = (\ell - p_2 - p_3)^2 - M_W^2$. The factor of 2 arises from the contributions of both the counterclockwise and clockwise W boson loops, which are identical. The momentum conservation implies

$$\ell_2 = \ell - p_2; \ell_3 = \ell - p_2 - p_3; p_1 = p_2 + p_3. \quad (3.6)$$

On the other hand, the bubble loop contribution shown in the case (c) of Figure (3.1) reads:

$$\begin{aligned} i\mathcal{M}^{(c)}(H \rightarrow Z\gamma) &= \frac{e^3 c_W M_W}{s_W^2} \varepsilon_{\lambda_2}^{\mu*}(p_2) \varepsilon_{\lambda_3}^{\nu*}(p_3) \\ &\times \int \frac{d^4 \ell}{(2\pi)^4} \frac{\mathcal{B}_{\mu\nu\rho\sigma} g_{\alpha\beta}}{D_0 D_2} \left(g^{\alpha\rho} - \frac{\ell^\alpha \ell^\rho}{M_W^2} \right) \left(g^{\sigma\beta} - \frac{\ell_3^\sigma \ell_3^\beta}{M_W^2} \right). \end{aligned} \quad (3.7)$$

The amplitude of interest is the sum of the triangle (3.5) and bubble (3.7) loop contributions. The object $\mathcal{B}_{\mu\nu\sigma\rho}$ is defined in (2.27) and $\mathcal{V}_{\mu\nu\rho}$ is an antisymmetric tensor defined in (2.26).

3.3 Tensor Reduction

The approach for calculating the amplitude is to solve the loop integral. Although it is finite in 4 dimensions, it is useful to use dimensional regularization by expanding the loop integral to d dimensions, and applying the Passarino-Veltman tensor reduction. This results in a linear combination of tensor structures that depend only on the external momenta.

The decay amplitude, $\mathcal{M} = \mathcal{M}^{(b)} + \mathcal{M}^{(c)}$, can be decomposed as follows [10]:

$$i\mathcal{M} = \varepsilon_{\mu}^{\lambda_2*}(p_2) \varepsilon_{\nu}^{\lambda_3*}(p_3) [F_0 g^{\mu\nu} + F_1 p_2^{\mu} p_3^{\nu} + F_2 p_2^{\mu} p_2^{\nu} + F_3 p_3^{\mu} p_3^{\nu} + F_4 p_3^{\mu} p_2^{\nu}]. \quad (3.8)$$

By means of the Ward-Takahashi identity and the condition $\varepsilon_{\lambda_3}^*(p_3) \cdot p_3 = 0$, only F_4 contributes, so the amplitude depends on only one form factor.

$$i\mathcal{M} = \varepsilon_{\mu}^{\lambda_2*}(p_2) \varepsilon_{\nu}^{\lambda_3*}(p_3) F_4 [-(p_2 \cdot p_3) g^{\mu\nu} + p_3^{\mu} p_2^{\nu}]. \quad (3.9)$$

Using the decay width formula (2.29) and the simplified expression for the amplitude (3.9), the decay width can be expressed, via the procedure explained in Appendix (B.5), as follows [12, 48, 49]:

$$\Gamma(H \rightarrow Z\gamma) = \frac{M_H^3}{32\pi} \left(1 - \frac{M_Z^2}{M_H^2}\right)^3 |F_4|^2. \quad (3.10)$$

This implies that it is sufficient to compute only the form factor F_4 . Therefore, we can focus specifically on the tensor structures contributing to $g^{\mu\nu}$ or to $p_3^{\mu} p_2^{\nu}$.

To compute F_4 , we perform the loop integral using dimensional regularization, a well-established technique in which space-time and momentum space dimensions are generalized. In this approach, tensor objects containing momenta and metric tensors are handled by setting the loop integration to d -dimensions [50]:

$$\int \frac{d^4 \ell}{(2\pi)^4} \rightarrow \mu^{4-d} \int \frac{d^d \ell}{(2\pi)^d}, \quad (3.11)$$

where μ is the scale of dimensional regularization, ensuring the form factor F_4 has the correct mass dimension. This process involves reducing the tensor structures and

systematically solving the scalar integrals. We focus exclusively on contributions to $p_3^\mu p_2^\nu$, since this tensor structure explicitly contains that form factor in (3.9). With this in mind, we expand the numerator in the loop integrals of (3.7) and (3.5) and, due to the Ward-Takahashi identity and the transversality of the polarization vectors, we retain only these tensor structures in the loop integral: $\ell^\mu \ell^\nu$, $p_3^\mu \ell^\nu$, $\ell^\mu p_2^\nu$ and $p_3^\mu p_2^\nu$.

Thus, through algebraic simplifications and the procedure explained in Appendix (B.6), we obtain the tensor-reduced amplitude [51], which aligns with the result in [33].

$$\begin{aligned}
i\mathcal{M}^{(b)} \rightarrow & \left(\frac{e^3 c_W M_W}{s_W^2} \right) \mu^{4-d} \int \frac{d^d \ell}{(2\pi)^d} \frac{\varepsilon_{\lambda_2}^{\mu*}(p_2) \varepsilon_{\lambda_3}^{\nu*}(p_3)}{M_W^4} \\
& \times \left\{ \ell_\mu \ell_\nu \left[\frac{1}{D_2} + \frac{1}{D_0} - \frac{2M_Z^2}{D_1 D_2} - \frac{2M_W^2 + M_H^2}{D_0 D_2} \right. \right. \\
& \quad \left. \left. - \frac{8(d-2)M_W^4 + 4M_W^2 M_H^2 + 8M_W^2 - 2M_H^2 M_Z^2 - 4M_W^2 M_Z^2}{D_0 D_1 D_2} \right] \right. \\
& \quad + \ell_\mu p_{2\nu} \left[-\frac{1}{D_2} - \frac{1}{D_0} + \frac{2M_Z^2}{D_1 D_2} + \frac{8M_W^2 + M_H^2}{D_0 D_2} - \frac{4M_W^2 - M_Z^2}{D_0 D_1} \right. \\
& \quad \left. \left. + \frac{8(d-2)M_W^4 + 4M_W^2 M_H^2 + 8M_W^2 - 2M_H^2 M_Z^2 - 4M_W^2 M_Z^2}{D_0 D_1 D_2} \right] \right. \\
& \quad + p_{3\mu} \ell_\nu \left[\frac{4M_W^2}{D_1 D_2} - \frac{6M_W^2}{D_0 D_2} \right] \\
& \quad \left. + p_{3\mu} p_{2\nu} \left[-\frac{4M_W^2}{D_1 D_2} - \frac{2M_W^2}{D_0 D_2} - \frac{16M_W^4 - 4M_W^2 M_Z^2}{D_0 D_1 D_2} \right] \right\}, \tag{3.12}
\end{aligned}$$

and the bubble loop contribution is expressed as follows,

$$\begin{aligned}
i\mathcal{M}^{(c)} \rightarrow & \left(\frac{e^3 c_W M_W}{s_W^2} \right) \mu^{4-d} \int \frac{d^d \ell}{(2\pi)^d} \frac{\varepsilon_{\lambda_2}^{\mu*}(p_2) \varepsilon_{\lambda_3}^{\nu*}(p_3)}{M_W^4} \\
& \times \left\{ \ell_\mu \ell_\nu \left[-\frac{1}{D_2} - \frac{1}{D_0} + \frac{2M_W^2 + M_H^2}{D_0 D_2} \right] + \ell_\mu p_{2\nu} \left[\frac{1}{2D_2} + \frac{1}{2D_0} - \frac{2M_W^2 + M_H^2}{2D_0 D_2} \right] \right. \\
& \quad \left. + p_{3\mu} \ell_\nu \left[\frac{1}{2D_2} + \frac{1}{2D_0} - \frac{2M_W^2 + M_H^2}{2D_0 D_2} \right] + p_{3\mu} p_{2\nu} \left[\frac{2M_W^2}{D_0 D_2} \right] \right\}. \tag{3.13}
\end{aligned}$$

Notably, some terms, even divergent terms, will cancel in the sum $\mathcal{M} = \mathcal{M}^{(b)} + \mathcal{M}^{(c)}$. In particular, the $\ell_\mu \ell_\nu$ terms in both contributions contain divergent terms (terms with the same denominators) that will cancel out in the sum. The tensor-reduced amplitude

of the process $H \rightarrow Z\gamma$ reads

$$\begin{aligned}
i\mathcal{M}(H \rightarrow Z\gamma) &= i\mathcal{M}^{(b)} + i\mathcal{M}^{(c)} \rightarrow \left(\frac{e^3 c_W M_W}{s_W^2} \right) \mu^{4-d} \int \frac{d^d \ell}{(2\pi)^d} \frac{\varepsilon_{\lambda_2}^{\mu*}(p_2) \varepsilon_{\lambda_3}^{\nu*}(p_3)}{M_W^4} \\
&\times \left\{ \ell_\mu \ell_\nu \left[-\frac{2M_Z^2}{D_1 D_2} - \frac{8(d-2)M_W^4 + 4M_W^2 M_H^2 + 8M_W^2 - 2M_H^2 M_Z^2 - 4M_W^2 M_Z^2}{D_0 D_1 D_2} \right] \right. \\
&+ \ell_\mu p_{2\nu} \left[-\frac{1}{2D_2} - \frac{1}{2D_0} + \frac{2M_Z^2}{D_1 D_2} + \frac{14M_W^2 + M_H^2}{2D_0 D_2} - \frac{4M_W^2 - 2M_Z^2}{D_0 D_1} \right. \\
&+ \left. \left. \frac{8(d-2)M_W^4 + 4M_W^2 M_H^2 + 8M_W^2 - 2M_H^2 M_Z^2 - 4M_W^2 M_Z^2}{D_0 D_1 D_2} \right] \right. \\
&+ p_{3\mu} \ell_\nu \left[\frac{1}{2D_2} + \frac{1}{2D_0} + \frac{4M_W^2}{D_1 D_2} - \frac{14M_W^2 + M_H^2}{2D_0 D_2} \right] \\
&+ \left. \left. p_{3\mu} p_{2\nu} \left[-\frac{4M_W^2}{D_1 D_2} - \frac{16M_W^2 - 4M_W^2 M_Z^2}{D_0 D_1 D_2} \right] \right\}. \tag{3.14}
\end{aligned}$$

At this stage, the tensor-reduced amplitude is represented as a sum of contributing tensor structures with their respective form factors. These form factors no longer contain loop momenta in the numerator but have denominators corresponding to various well-defined n -point tensor integrals, allowing them to be expressed in terms of Passarino-Veltman functions [52].

By analyzing the expression in (3.14) and considering the number of numerators (D_i) as well as the indices of the loop momenta, we identify the Passarino-Veltman functions contributing to the amplitude. These include tensor functions of the form $B_{\mu\nu}^{(ij)}$, $C_{\mu\nu}^{(ijk)}$, $A_\mu^{(i)}$, $B_\mu^{(ij)}$, and $C_\mu^{(ijk)}$, also there are irreducible scalar integrals B_0 and C_0 . All tensor Passarino-Veltman functions can be systematically reduced to scalar integrals, as detailed in Appendix (B.7). The resulting simplifications yield the following expression:

$$\begin{aligned}
F_4 &= \frac{-i\pi^2 c_W e^3}{M_W^3 s_W^2 (M_H^2 - M_Z^2)^2} \left\{ M_Z^2 [M_H^2 (M_Z^2 - 2M_W^2) + 2M_W^2 (M_Z^2 - 6M_W^2)] \right. \\
&\times [B_0(M_H^2, M_W^2, M_W^2) - B_0(M_Z^2, M_W^2, M_W^2)] \\
&- 2M_W^2 (M_H^2 - M_Z^2) [M_H^2 (M_Z^2 - 6M_W^2) + 12M_W^4 + 6M_W^2 M_Z^2 - 2M_Z^4] \\
&\times C_0(0, M_H^2, M_Z^2, M_W^2, M_W^2, M_W^2) \left. \right\}. \tag{3.15}
\end{aligned}$$

With this result, the elimination of divergences becomes evident due to the difference

between the two B_0 functions. The next step involves expanding the Passarino-Veltman functions into a Laurent series [53] by setting $d = 4 - 2\varepsilon$. Only three irreducible Passarino-Veltman functions, detailed in Appendix (B.7), must be evaluated. The algebraic simplifications of the tensor-reduced amplitude were performed using FeynCalc, while the expansion of the Passarino-Veltman functions was carried out using FeynHelpers in conjunction with Package-X [54]. Furthermore, the final result can be expressed in terms of arcsines [55, 56] as shown in Appendix (B.7), with identities found in [57], as demonstrated in previous studies [10–12, 58]. The final result for the contributing form factor can be presented as in [33],

$$F_4 = \frac{ie^3 c_W}{16\pi^2 M_W s_W^2} \left\{ \left[5 + \frac{2}{\tau_H} - \left(1 + \frac{2}{\tau_H} \right) \frac{s_W^2}{c_W^2} \right] I_1(\tau_H, \tau_Z) - \left(12 - \frac{4s_W^2}{c_W^2} \right) I_2(\tau_H, \tau_Z) \right\}, \quad (3.16)$$

with $\tau_i = \frac{4M_W^2}{M_i^2}$, in order to make the expression compact, the following functions are defined

$$I_1(\tau_i, \tau_j) = \frac{\tau_i \tau_j}{2(\tau_i - \tau_j)} + \frac{\tau_i^2 \tau_j^2}{2(\tau_i - \tau_j)^2} [f(\tau_i) - f(\tau_j)] + \frac{\tau_i^2 \tau_j}{(\tau_i - \tau_j)^2} [g(\tau_i) - g(\tau_j)], \quad (3.17)$$

$$I_2(\tau_i, \tau_j) = -\frac{\tau_i \tau_j}{2(\tau_i - \tau_j)} [f(\tau_i) - f(\tau_j)], \quad (3.18)$$

with the functions [10] defined in the kinematic region of our interest ($4M_W^2 \geq M_H^2$):

$$g(\tau_i) = \sqrt{\tau_i - 1} \arcsin\left(\frac{1}{\sqrt{\tau_i}}\right), \quad \forall \tau_i \geq 1. \quad (3.19)$$

$$f(\tau_i) = \arcsin^2\left(\frac{1}{\sqrt{\tau_i}}\right), \quad \forall \tau_i \geq 1. \quad (3.20)$$

3.4 Partial Decay Width

Using the expression for the contributing form factor (3.16) and the reduced form of the decay width (3.10), we can evaluate the decay width as a function of the Higgs boson mass. To perform this calculation, we substitute the numerical values of the relevant physical quantities: $\frac{1}{137} = \frac{e^2}{4\pi}$, $M_W \approx 80.36$ GeV, $M_Z \approx 91.8$ GeV, and the sine

of the Weinberg angle $s_W^2 \approx 0.2$ [59]. The decay width of the process $H \rightarrow Z\gamma$ was found to be

$$\Gamma(H \rightarrow Z\gamma) = 8.33 \text{ KeV}. \quad (3.21)$$

In this case, we will evaluate the branching ratio, a physical observable measured by the ATLAS and CMS collaborations:

$$\text{BR}(H \rightarrow Z\gamma) = \frac{\Gamma(H \rightarrow Z\gamma)}{\Gamma_H} = 2.25 \times 10^{-3}, \quad (3.22)$$

which is of the same order of magnitude as the experimental result given in (2.30), although a notable discrepancy $\Delta_{\text{BR}} = 1.15 \times 10^{-3}$ in the exact value remains.

The total decay width of the Higgs boson, Γ_H , is reported by the Particle Data Group as $\Gamma_H = 3.7 \times 10^{-3} \text{ GeV}$. Consequently, the branching ratio for the Higgs boson decaying into a Z boson and a photon is evaluated within the mass range $90 < M_H < 150 \text{ GeV}$. The plot of the branching ratio in SM at LO as a function of the Higgs boson mass was generated using an integrated workflow between Wolfram Mathematica and Python.

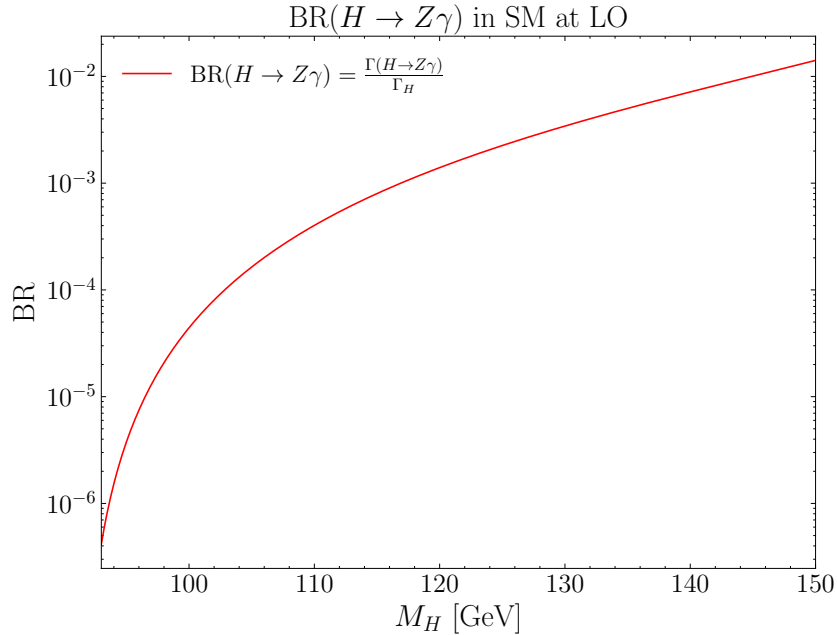


Figure 3.3: Branching ratio of the $H \rightarrow Z\gamma$ decay process in SM as a function of the Higgs boson mass.

In Figure (3.3), we observe that the branching ratio for this process is very small, with values on the order of at most 10^{-2} for a Higgs boson mass of $M_H = 150$ GeV. For a Higgs boson with a mass of $M_H = 125$ GeV, the branching ratio is on the order of 10^{-3} , as shown in Figure (2.1a). However, this value presents a discrepancy with the experimental branching ratio, suggesting that corrections are needed, either from higher-order contributions (NNLO) within the SM or from BSM physics.

3.5 Computational Systematization

A Wolfram Mathematica package, HZAprogram.wls, was developed to automate the computation of the decay width for $H \rightarrow Z\gamma$ and derive its analytical expression. This expression is then evaluated in Python to compute the branching ratio as a function of the Higgs boson mass. This section outlines the general procedure and highlights key code components used to obtain analytical and numerical results. The package is available in the following GitHub repository: [GitHub - fisicateoricaUDP - HiggsDecays](#).

First, the corresponding diagrams were generated using the FeynArts package and then translated into analytical expressions compatible with FeynCalc. The amplitude was computed in the unitary gauge and in d dimensions, resulting in three 1PI diagrams involving the W boson. These diagrams were summed and subsequently contracted. The polarization vectors were truncated, as the primary focus was to identify the contributing form factor F_4 . First, the packages are imported.

```
#!/usr/bin/env wolframscript
(* ::Package:: *)

If[ $FrontEnd == Null ,
    $FeynCalcStartupMessages = False;
    Print["Computation"]; ];
Off[FrontEndObject::notavail]
$LoadAddOns={"FeynHelpers","FeynArts"};
<<FeynCalc';
$FAVerbose =0;
Off[Paint::nolevel];
```

The amplitude in d-dimensions is generated using FeynArts. First, the topologies are

created with the "CreateTopologies" function, producing 1PI topologies by specifying "Internal" in the "ExcludeTopologies" option. The unitary gauge is then set using the "InitializeModel" and "GenericModel" functions. Irrelevant fields, such as fermions and gluons, are excluded by using the "exclusions" option. Consequently, the desired Feynman diagrams are obtained.

```

Topol = CreateTopologies[1,1->2,ExcludeTopologies->{Internal},Adjacencies
->{{3,4}}];
exclusions = {S[2],S[3],
F[1,{_}],F[2,{_}],F[3,{_}],F[4,{_}],
U[1 | 2 | 3 | 4 | 5 ],
V[5]};

InitializeModel[{SM, UnitarySM}, GenericModel -> {Lorentz, UnitaryLorentz
}];
DiagF =InsertFields[Topol,{S[1]}->{V[2],V[1]},Model -> {SM, UnitarySM},
GenericModel -> {Lorentz, UnitaryLorentz},
InsertionLevel -> {Particles},
ExcludeParticles->exclusions];

```

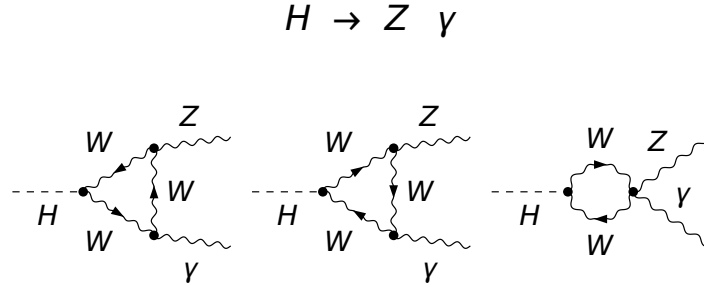


Figure 3.4: Feynman diagrams for W boson loop contributions to $H \rightarrow Z\gamma$ generated by FeynArts.

As previously discussed, three 1PI diagrams are generated in the unitary gauge using "CreateFeynAmp", as there are no unphysical contributions in this gauge. Furthermore, in the Wolfram Engine, it is necessary to export the diagrams to a PDF for visualization using the "Export" function. Subsequently, the expression is converted into FeynCalc language with the "FCFAConvert" function.

```

SetOptions[Paint, ColumnsXRows->{3,1}, FieldNumbers->True, Numbering->False
];
Amp=CreateFeynAmp[DiagF, PreFactor -> 1, Truncated->True, GaugeRules -> {
  FAGaugeXi[W | Z] -> Infinity}];
AmpFC=FCFAConvert[Amp, ChangeDimension->D,
  IncomingMomenta->{p1},
  OutgoingMomenta->{p2, p3},
  LoopMomenta->{1}] // Simplify ;

```

Now, fixing the kinematics to reduce the computations and find the useful Passarino-Veltman functions is important. Then, we sum up the three diagrams and perform the Lorentz contractions.

```

FCClearScalarProducts[];
SPD[p3, p3]= 0;
SPD[p2, p2]=mz^2;
SPD[p1, p1]=mh^2;
SPD[p2, p3]= (mh^2-mz^2)/2;
Ampw =Total[Amp2]//Simplify // Contract //FCE ;

```

The tensor reduction is applied to the amplitude using the "TID" function. Next, the denominators are analyzed to identify the irreducible integrals, specifically the Passarino-Veltman functions, which are stored in the "ChangeDenToPV" list. These functions are then expanded into a Laurent series using "PaXEvaluate," provided by Package-X in FeynHelpers, and the results are saved in the "ChangePVToLaurent" list. Subsequently, the scalar integrals are evaluated.

```

ChangeDenToPV = {expression_>TID[expression, l, ToPaVe -> True]};
Redu = TID[Ampw, l]//Simplify;
PVfunctions = Cases[Redu, FeynAmpDenominator[___], Infinity]/(I*Pi^2) /.
  ChangeDenToPV // DeleteDuplicates;
ChangePVToLaurent = Table[(PVfunctions[[i]] -> PaXEvaluate[#,
  PaXImplicitPrefactor->1/(2*Pi)^(4-2*Epsilon)]) & [PVfunctions[[i]]], {i
  , 1, Length[PVfunctions]}];
Redu1 = ToPaVe[Redu, l]//FCE;

```

Now, we isolate the form factor F_4 using the "Coefficient" function, which extracts the coefficients of a specified term—in this case, the tensor structure $p_3^\mu p_2^\nu$. We set $d = 4 - 2\epsilon$ and perform simplifications. This leads to a finite result, after which we convert the objects in the expression to four dimensions using the "ChangeDimension" function and impose the kinematics. Next, we calculate the squared modulus of F_4 , and then input it into the function defined in the code, as outlined in (3.10), replacing the numerical values of the physical quantities. This yields an expression with the Higgs boson mass as the input variable.

```

ReduF4 = Coefficient[Redu1,FVD[p3, \[Mu]]*FVD[p2, \[Nu]]] //Simplify;
RedAmp = (ReduF4 /. ChangePVToLaurent) // FCReplaceD[#, D -> 4 - 2*
  Epsilon] & // Series[#, {Epsilon, 0, 0}] & // Normal //Simplify //FCE;
F4 = RedAmp// ChangeDimension[#, 4] &
SP[p3,p3]=0;
SP[p2,p3]= (mh^2-mz^2)/2;
SP[p2,p2]= mz^2;
SF4 = F4*(ComplexConjugate[F4]) //Simplify // FCE ;
DWidth = mh^3/(32*Pi)*(1- mz^2/mh^2)^3*SF4/.mw->80.36/.mz->91.18/.SW->
  Sqrt[0.2]/.CW->Sqrt[0.8]/.e->Sqrt[4*Pi/137]//Simplify;

```

This expression is exported to a text file, and then, with this result, we incorporate it into a Python library used in the "NumericalDW.py". This program evaluates the branching ratio of $H \rightarrow Z\gamma$ as a function of the Higgs boson mass and generates a plot, shown in Figure (3.3), using the SciencePlots library [60].

```

ReduF4 = Coefficient[Redu1,FVD[p3, \[Mu]]*FVD[p2, \[Nu]]] //Simplify;
RedAmp = (ReduF4 /. ChangePVToLaurent) // FCReplaceD[#, D -> 4 - 2*
  Epsilon] & // Series[#, {Epsilon, 0, 0}] & // Normal //Simplify //FCE;
F4 = RedAmp// ChangeDimension[#, 4] &
SP[p3,p3]=0;
SP[p2,p3]= (mh^2-mz^2)/2;
SP[p2,p2]= mz^2;
SF4 = F4*(ComplexConjugate[F4]) //Simplify // FCE ;
DWidth = mh^3/(32*Pi)*(1- mz^2/mh^2)^3*SF4/.mw->80.36/.mz->91.18/.SW->
  Sqrt[0.2]/.CW->Sqrt[0.8]/.e->Sqrt[4*Pi/137]//Simplify;

```

We wrote a Linux bash script called "manager" that transfers the final expression generated by the code "HZAprogram.wls" into the "HZYDecaywidth.py" program.

```
#!/bin/bash
(sed '6 r HZADwidth' HZDW.py) > HZYDecaywidth.py;
cat HZYDecaywidth.py;
python3 NumericalDW.py;
```

The package to automate the computation includes a text file named "README", which explains the steps required to execute the scripts and outlines the dependencies necessary to function correctly.

4. Conclusions

In this work, the analytical expression of the decay width of the Higgs boson decaying into a Z boson-photon pair, at leading order in the Standard Model, was derived. The computation of the dominant contributions has included the evaluation of one-loop vertex diagrams with a virtual W boson field in the loop. We have verified that this contribution is finite for different gauges. Using dimensional regularization and the Passarino-Veltman approach, we have isolated the UV divergences of each diagram and obtained the decay width in terms of just one finite form factor, as is required by the Ward-Takahashi identities. It was found that the unitary gauge provides a more efficient computational approach, as it restricts the calculation to only 1PI contributions with neither ghost nor Goldstone boson fields in the loop, reducing the number of diagrams to be evaluated. In contrast, in the general R_ξ gauge, virtual ghost and Goldstone bosons must be taken into account, and non-1PI diagrams must be included to cancel the UV divergences from the 1PI diagrams. In particular, in the Yennie gauge $\xi = -3$, the coefficient of the simple pole is directly zero. The finite part of non-1PI diagrams does not make contributions to the corresponding form factor and, therefore, to the decay width, as expected from the Cutkosky cutting rules. Given the complexity of calculating this observable at one-loop level, automated methods were employed. The systematization process was implemented using the language of Wolfram Mathematica, with the help of the packages FeynArts, FeynCalc and FeynHelpers. A numerical estimation of the resulting partial decay width was done with a script in Python. The partial decay width was determined to be 8.33 keV, with a branching ratio of approximately 2.25×10^{-3} , indicating a discrepancy of 1.15×10^{-3} compared to the experimental value. This result exhibits the well known discrepancy with the experimental observations at LHC. The SM prediction at LO indicates a numerical excess, which could potentially be attributed to invisible effects coming from BSM physics. The NLO do not solve the discrepancy, and in particular, the EW NLO contribution is negative and about 10 times larger than the NLO QCD correction. It is important therefore to estimate NNLO corrections in the EW sector for this decay channel, but also to study the effects of phenomenologically well motivated BSM scenarios. Furthermore, future experimental measurements of this decay process will be crucial

for validating and refining the theoretical predictions.

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Appendix A

Notation and Conventions

We are dealing with the natural units

$$\hbar = c = 1, \quad (\text{A.1})$$

therefore

$$[\text{length}] = [\text{time}] = [\text{mass}]^{-1} = [\text{Energy}]^{-1} \quad (\text{A.2})$$

The mostly negative signature of the metric tensor is used,

$$g_{\mu\nu} = g^{\mu\nu} = \text{diag}(1, -1, -1, -1). \quad (\text{A.3})$$

Furthermore, the 4-vectors are

$$A^\mu = (A^0, \vec{A}) = (A^0, A^1, A^2, A^3), \quad A_\mu = (A^0, -\vec{A}). \quad (\text{A.4})$$

The inner product convention between two functions is defined as

$$(A, \Delta B) = \int d^4x d^4y A^i(x) \Delta_{ij}(x-y) B^j(y), \quad (\text{A.5})$$

with an operator Δ . In this case, ij are collective indices like adjoint or Lorentz. The convention for summation for 4-vectors is defined for two repeated indices.

$$A \cdot B = A_\mu B^\mu = A^\mu B_\mu = g_{\mu\nu} A^\mu B^\nu = A^0 B^0 - \vec{A} \cdot \vec{B} = A^0 B^0 - A^i B^i, \quad (\text{A.6})$$

where Greek indices are fixed for Lorentz indices and latin indices are fixed for space indices. The 4-derivative is

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial t}, \vec{\nabla} \right), \quad \partial^\mu = \frac{\partial}{\partial x_\mu} = \left(\frac{\partial}{\partial t}, -\vec{\nabla} \right), \quad (\text{A.7})$$

and the d'Alembert operator is

$$\square = \partial^\mu \partial_\mu = \frac{\partial^2}{\partial t^2} - \nabla^2. \quad (\text{A.8})$$

The Levi-Civita tensor is

$$\varepsilon^{\mu\nu\rho\sigma} = -\varepsilon_{\mu\nu\rho\sigma} = \begin{cases} 1 & , \text{for odd permutations of the indices.} \\ -1 & , \text{for even permutations of the indices.} \\ 0 & , \text{otherwise.} \end{cases} \quad (\text{A.9})$$

The Green function $G^{ij}(x-y)$ is defined as

$$\Delta_{kj}G^{ij}(x-y) = -i\delta_{ik}\delta^{(4)}(x-y). \quad (\text{A.10})$$

Polarization vectors are defined to be transversal

$$p^\mu \varepsilon_\mu^\lambda(p) = 0, \quad (\text{A.11})$$

where λ is the index of the polarization state. The polarization vectors satisfies the completeness relation for massive case

$$\sum_\lambda \varepsilon_\lambda^{\mu*}(p) \varepsilon_\lambda^\nu(p) = -g^{\mu\nu} + \frac{p^\mu p^\nu}{M^2}, \quad (\text{A.12})$$

and for massless case, we can write

$$\sum_\lambda \varepsilon_\lambda^{\mu*}(p) \varepsilon_\lambda^\nu(p) \rightarrow -g^{\mu\nu}. \quad (\text{A.13})$$

We have the convention for Passarino-Veltman function:

$$i\pi^2 F_{\mu\dots\rho}^{(ij\dots k)} = \mu^{4-d} \int \frac{d^d\ell}{(2\pi)^d} \frac{\ell_\mu \dots \ell_\rho}{D_i D_j \dots D_k}, \quad (\text{A.14})$$

where, $p_{(1)} = p_2$, $p_{(2)} = p_3$, $p_{(0)} = 0$. The irreducible scalar integrals for $m_1 = m_2 = m_3 = m = M_W$ read:

$$i\pi^2 A_{0,\mu}^{(i)} = i\pi^2 A_{0,\mu}(p_{(i)}^2, m^2) = \mu^{4-d} \int \frac{d^d\ell}{(2\pi)^d} \frac{\{1, \ell_\mu\}}{D_i}, \quad (\text{A.15})$$

$$i\pi^2 B_{0,\mu,\mu\nu}^{(ij)} = i\pi^2 B_{0,\mu,\mu\nu}(p_{(i)}^2, p_{(j)}^2, m^2, m^2) = \mu^{4-d} \int \frac{d^d\ell}{(2\pi)^d} \frac{\{1, \ell_\mu, \ell_\mu \ell_\nu\}}{D_i D_j}, \quad (\text{A.16})$$

$$i\pi^2 B_{0,\mu,\mu\nu}^{(0j)} = i\pi^2 B_{0,\mu,\mu\nu}(p_{(j)}^2, m^2, m^2) = \mu^{4-d} \int \frac{d^d\ell}{(2\pi)^d} \frac{\{1, \ell_\mu, \ell_\mu \ell_\nu\}}{D_0 D_j}, \quad (\text{A.17})$$

$$i\pi^2 C_{0,\mu,\mu\nu}^{(012)} = i\pi^2 C_{0,\mu,\mu\nu}(p_3^2, p_1^2, p_2^2, m^2, m^2, m^2) = \mu^{4-d} \int \frac{d^d\ell}{(2\pi)^d} \frac{\{1, \ell_\mu, \ell_\mu \ell_\nu\}}{D_0 D_1 D_2}. \quad (\text{A.18})$$

Appendix B

Additional Calculations

B.1 Ghost Propagator

Ghosts are useful for restoring gauge invariance in the theory. From Grassmann algebra, we know that the determinant of the Jacobian allows for a path integral representation using Grassmann fields [30]:

$$\text{Det} \left[\frac{\delta F(x)}{\delta \theta(y)} \right] \propto \int \mathcal{D}\bar{c} \mathcal{D}c \, e^{i \left(\bar{c}, \frac{\delta F(x)}{\delta \theta(y)} c \right)}, \quad (\text{B.1})$$

where $\bar{c}$ and c are the anti-ghost and ghost fields, respectively; $F(x)$ is the gauge function, and $\theta(y)$ is the gauge parameter. We can choose a specific gauge function for virtual W bosons, allowing us to fix one of the infinitely many possible equivalent states. Therefore,

$$C^\pm = \partial^\mu W_\mu^\pm \mp i M_W \xi G^\pm, \quad (\text{B.2})$$

because the Lagrangian is invariant under gauge transformations. In the case of the W boson and Goldstone boson,

$$W_\mu^\pm(x) \rightarrow W_\mu^\pm(x) + \partial_\mu \delta\theta^\pm \mp i \frac{e}{s_W} \left[W_\mu^\pm (s_W \delta\theta^A - c_W \delta\theta^Z) - (s_W A_\mu - c_W Z_\mu) \delta\theta^\pm \right], \quad (\text{B.3})$$

$$G^\pm \rightarrow G^\pm \mp i e G^\pm \left[\delta\theta^A + \frac{s_W^2 - c_W^2}{2c_W s_W} \delta\theta^Z \right] \pm \frac{i e}{2s_W} [v + H \pm i G] \delta\theta^\pm, \quad (\text{B.4})$$

where $\delta\theta^i$ is the gauge parameter of the transformation for each field Φ^i . Thereby, the gauge Jacobian of the transformation reads

$$\begin{aligned} C^\pm(x) &\rightarrow \partial^\mu W_\mu^\pm(x) + \square \delta\theta^\pm(x) \mp i \frac{e}{s_W} \left[\partial^\mu W_\mu^\pm(x) (s_W \delta\theta^A - c_W \delta\theta^Z) - \partial^\mu (s_W A_\mu - c_W Z_\mu) \delta\theta^\pm(x) \right] \\ &\mp \left[i M_W \xi G^\pm(x) \mp i^2 M_W \xi e G^\pm(x) \left[\delta\theta^A(x) + \frac{s_W^2 - c_W^2}{2c_W s_W} \delta\theta^Z(x) \right] \pm \frac{i^2 M_W \xi e}{2s_W} [v + H(x) \pm i G(x)] \delta\theta^\pm(x) \right], \end{aligned} \quad (\text{B.5})$$

thereby, the Jacobian reads

$$\frac{\delta C^\pm(x)}{\delta \theta^\pm(y)} = \delta^{(4)}(x-y) \left\{ \square \pm i \frac{e}{s_W} \partial^\mu (s_W A_\mu - c_W Z_\mu) \mp \left[\pm \frac{i^2 M_W \xi e}{2s_W} [v + H(x) \pm i G(x)] \right] \right\}. \quad (\text{B.6})$$

The Ghost action can be obtained

$$\text{Det} \left[\frac{\delta C^\pm(x)}{\delta \theta^\pm(y)} \right] = \int \mathcal{D}\bar{c} \mathcal{D}c \, e^{i \int d^4x \, \mathcal{L}_{\text{gh}}}, \quad (\text{B.7})$$

$$S_{\text{gh}} = \int d^4x \, \bar{c} \left\{ \square \pm i \frac{e}{s_W} \partial^\mu (s_W A_\mu - c_W Z_\mu) \mp \left[\pm \frac{i^2 M_W \xi e}{2s_W} [v + H(x) \pm i G(x)] \right] \right\} c. \quad (\text{B.8})$$

In order to obtain the ghost propagator, we take a look on the bilinear ghost terms:

$$S_{\text{gh-prop}} = \int d^4x \bar{c} \left(\square + \xi M_{\text{gh}}^2 \right) c, \quad (\text{B.9})$$

thereby, the ghost propagator is the inverse of the Klein-Gordon operator

$$\tilde{\Delta}_{\text{gh}}(p) = \frac{i}{p^2 - \xi M_{\text{gh}}^2}, \quad (\text{B.10})$$

furthermore, in the unitary gauge ($\xi \rightarrow \infty$), the ghost propagator vanishes, which implies that ghosts in the unitary gauge are static and do not contribute.

B.2 Obtaining the Green Function

The generating functional (3.3) can be written as

$$Z[J] = Z[0] e^{i \int d^4x \mathcal{L}_{\text{int}}\left(\frac{\delta}{\delta J(x)}, \frac{\delta}{\delta j(x)}\right)} e^{(J_+, \Delta_W J_-)} e^{\frac{1}{2}(J_0, \Delta_Z J_0)} e^{\frac{1}{2}(J, \Delta_\gamma J)} e^{(j, \Delta_F j)}. \quad (\text{B.11})$$

The Green function from the expansion of the generating functional (B.11) reads

$$\begin{aligned} G(x_1, x_2, x_3) &= \frac{\delta^{(3)}}{\delta j(x_1) \delta J_0(x_2)^\mu \delta J(x_3)^\nu} \int d^4x \sum_{k=0}^{\infty} \frac{i^k}{k!} \left[\frac{e M_W}{s_W} H W_\mu^+ W_-^\mu + i e \left(W_{\mu\nu}^+ W_-^\mu A^\nu - W_{\mu\nu}^- W_+^\mu A^\nu + F_{\mu\nu} W_+^\mu W_-^\nu \right) - \right. \\ &\quad \left. i \frac{e c_W}{s_W} \left(W_{\mu\nu}^+ W_-^\mu Z^\nu - W_{\mu\nu}^- W_+^\mu Z^\nu + Z_{\mu\nu} W_+^\mu W_-^\nu \right) \right] \Big|_{\Phi \rightarrow \frac{\delta}{\delta j \Phi}} \sum_{m=0}^{\infty} \frac{1}{m! 2^m} (J, \Delta_\gamma J)^m \sum_{n=0}^{\infty} \frac{1}{n! 2^n} (J_0, \Delta_Z J_0)^n \\ &\quad \times \sum_{\ell=0}^{\infty} \frac{1}{\ell! 2^\ell} (j, \Delta_F j)^\ell \sum_{N=0}^{\infty} \frac{1}{N!} (J_-, \Delta_W J_+)^N, \end{aligned} \quad (\text{B.12})$$

where J is the photon source, J_0 refers to the source corresponding to the neutral current (Z boson) and $J_\pm$ to the charged currents. Since we have three external source derivatives, we need the third-order term in the expansion of the interaction term; the only term that will be given a nonzero value is the term that contains the three bosons outside the loop, that is the term that includes the Higgs boson field, Z boson, and photon.

In the one loop diagram, obtained in the third perturbative order in the interaction term, $k = 3$, we get the first term on the remaining series for the sources ($n = m = \ell = 1, N = 3$)

$$\begin{aligned}
G(x_1, x_2, x_3) &= \frac{i^3}{3!} \frac{\delta^{(3)}}{\delta j(x_1) \delta J_0(x_2)^\mu \delta J(x_3)^\nu} \int d^4x \left[\frac{eM_W}{s_W} HW_\mu^+ W_-^\mu + i e \left(W_{\mu\nu}^+ W_-^\mu A^\nu - W_{\mu\nu}^- W_+^\mu A^\nu + F_{\mu\nu} W_+^\mu W_-^\nu \right) \right. \\
&\quad \left. - i \frac{e c_W}{s_W} \left(W_{\mu\nu}^+ W_-^\mu Z^\nu - W_{\mu\nu}^- W_+^\mu Z^\nu + Z_{\mu\nu} W_+^\mu W_-^\nu \right) \right] \Big|_{\Phi \rightarrow \frac{\delta}{\delta j \Phi}} \\
&\quad \times \frac{1}{2} (J, \Delta_\gamma J) \frac{1}{2} (J_0, \Delta_Z J_0) \frac{1}{2} (j, \Delta_F j) \frac{1}{3!} (J_-, \Delta_W J_+)^3.
\end{aligned} \tag{B.13}$$

Now, the nontrivial term in (B.13) is

$$\begin{aligned}
G^{(1)}(x_1, x_2, x_3) &= -i \frac{1}{3!} \frac{\delta^{(3)}}{\delta j(x_1) \delta J_Z(x_2)^\mu \delta J(x_3)^\nu} \int d^4x \left[\frac{eM_W}{s_W} HW_\mu^+ W_-^\mu \right. \\
&\quad \times (ie) \left(W_{\mu\nu}^+ W_-^\mu A^\nu - W_{\mu\nu}^- W_+^\mu A^\nu + F_{\mu\nu} W_+^\mu W_-^\nu \right) \\
&\quad \times \left(-i \frac{e c_W}{s_W} \right) \left(W_{\mu\nu}^+ W_-^\mu Z^\nu - W_{\mu\nu}^- W_+^\mu Z^\nu + Z_{\mu\nu} W_+^\mu W_-^\nu \right) \Big|_{\Phi \rightarrow \frac{\delta}{\delta j \Phi}} \\
&\quad \times \frac{1}{2} (J, \Delta_\gamma J) \frac{1}{2} (J_0, \Delta_Z J_0) \frac{1}{2} (j, \Delta_F j) \frac{1}{3!} (J_-, \Delta_W J_+)^3.
\end{aligned} \tag{B.14}$$

We found that the expression (B.14) is the nontrivial term from the interaction term. From (B.13), we first consider three fundamental terms: Higgs-W boson cubic term, W boson - Z boson cubic term and W boson - Photon cubic term, which contain each external line, and then mix them in order to form the W boson loop:

$$G^{(1)}(x_1, x_2, x_3) = i \frac{1}{3!} [G_{\text{Higgs}} G_Z G_\gamma] \frac{1}{3!} (J_-, \Delta_W J_+)^3, \tag{B.15}$$

where each contribution to the correlation function ($G_{\text{Higgs}}, G_Z, G_\gamma$) contains each external bosonic line. Thereby,

$$\begin{aligned}
G_{\text{Higgs}} &= \frac{1}{2} \int d^4x \frac{\delta}{\delta j(x_1)} \frac{eM_W}{s_W} HW_\mu^+ W_-^\mu(x) \Big|_{H \rightarrow \frac{\delta}{\delta j}, W \rightarrow \frac{\delta}{\delta J_W}} (j, \Delta_F j) \\
&= \frac{1}{2} \frac{eM_W}{s_W} g^{\mu\nu} (2) \int d^4x \frac{\delta}{\delta j(x_1)} HW_\mu^+ W_\nu^-(x) \Big|_{H \rightarrow \frac{\delta}{\delta j}, W \rightarrow \frac{\delta}{\delta J_W}} (j, \Delta_F j) \\
&= \frac{eM_W}{s_W} g^{\mu\nu} \int d^4x \frac{\delta}{\delta j(x_1)} \frac{\delta}{\delta j(x)} \frac{\delta}{\delta J_-(x)^\mu} \frac{\delta}{\delta J_+(x)^\nu} (j, \Delta_F j) \\
&= \frac{eM_W}{s_W} g^{\mu\nu} \int d^4x \frac{\delta}{\delta j(x_1)} \frac{\delta}{\delta j(x)} (j, \Delta_F j) \frac{\delta}{\delta J_-(x)^\mu} \frac{\delta}{\delta J_+(x)^\nu} \\
G_{\text{Higgs}} &= \frac{eM_W}{s_W} g^{\mu\nu} \int d^4x \Delta_F(x-x_1) \frac{\delta^{(2)}}{\delta J_-(x)^\mu \delta J_+(x)^\nu},
\end{aligned} \tag{B.16}$$

where the factor of 2 is due to the symmetry factor because of the reciprocity of the scalar Feynman propagator. This symmetry applies to the symmetric Green function and to neutral sources, such as in the case of Z . This symmetry reads

$$\begin{aligned}
\frac{\delta^{(2)}}{\delta J^i(x) \delta J^j(y)} (J, \Delta J) &= \frac{\delta^{(2)}}{\delta J^i(x) \delta J^j(y)} \int d^4 z d^4 w J^a(z) \Delta_{ab}(z-w) J^b(w) \\
&= \frac{\delta}{\delta J^i(x)} \int d^4 z d^4 w \left[J^a(z) \Delta_{ab}(z-w) \frac{\delta J^b(w)}{\delta J^j(y)} + \frac{\delta J^a(z)}{\delta J^j(y)} \Delta_{ab}(z-w) J^b(w) \right] \\
&= \frac{\delta}{\delta J^i(x)} \int d^4 z d^4 w \left[J^a(z) \Delta_{ab}(z-w) \delta^{(4)}(w-y) \delta_{jb} + \delta^{(4)}(z-y) \delta_{ja} \Delta_{ab}(z-w) J^b(w) \right] \\
&= \left[\int d^4 z \frac{\delta J^a(z)}{\delta J^i(x)} \Delta_{aj}(z-y) + \int d^4 w \Delta_{jb}(y-w) \frac{\delta J^b(w)}{\delta J^i(x)} \right] \\
&= \Delta_{ij}(x-y) + \Delta_{ji}(y-x) = 2\Delta_{ji}(y-x) \quad \because \Delta_{ji}(y-x) = \Delta_{ij}(x-y) = \Delta_{ji}(x-y).
\end{aligned} \tag{B.17}$$

Now, we form the trilinear terms for the gauge bosons:

$$G_Z = \frac{\delta}{\delta J_0(x_2)^\mu} \int d^4 x \left(-i \frac{e c_W}{s_W} \right) \left(W_{\mu'\nu'}^+ W_{-}^{\mu'} Z^{\nu'} - W_{\mu'\nu'}^- W_{+}^{\mu'} Z^{\nu'} + Z_{\mu'\nu'} W_{+}^{\mu'} W_{-}^{\nu'} \right) \Big|_{V \rightarrow \frac{\delta}{\delta J_V}} \frac{1}{2} (J_0, \Delta_Z J_0). \tag{B.18}$$

Note in one of the three cubic terms in G_Z :

$$\begin{aligned}
&\frac{\delta}{\delta J_0(x_2)^\mu} \left(W_{\mu'\nu'}^+ W_{-}^{\mu'} Z^{\nu'} \right) \Big|_{V \rightarrow \frac{\delta}{\delta J_V}} (J_0, \Delta_Z J_0) = \\
&g^{\mu'\rho} g^{\nu'\alpha} \frac{\delta}{\delta J_0(x_2)^\mu} \left(W_{\mu'\nu'}^+ \frac{\delta}{\delta J_+(x)^\rho} \frac{\delta}{\delta J_0(x)^\alpha} \right) (J_0, \Delta_Z J_0) = \\
&g^{\mu'\rho} g^{\nu'\alpha} \left(W_{\mu'\nu'}^+ \frac{\delta}{\delta J_+(x)^\rho} \right) \frac{\delta^{(2)}}{\delta J_0(x_2)^\mu \delta J_0(x)^\alpha} (J_0, \Delta_Z J_0) = \\
&g^{\mu'\rho} g^{\nu'\alpha} \left(W_{\mu'\nu'}^+ \frac{\delta}{\delta J_+(x)^\rho} \right) \Delta_Z(x_2 - x)_{\mu\alpha},
\end{aligned} \tag{B.19}$$

which implies a similar way to realize the second term in (B.18), now we try the term:

$$\begin{aligned}
&g^{\mu'\rho} g^{\nu'\alpha} Z_{\mu'\nu'} \frac{\delta^{(2)}}{\delta J_-(x)^\rho \delta J_+(x)^\alpha} \frac{\delta}{\delta J_0(x_2)^\mu} (J_0, \Delta_Z J_0) = \\
&\left[g^{\mu'\rho} g^{\nu'\alpha} \frac{\delta^{(2)}}{\delta J_-(x)^\rho \delta J_+(x)^\alpha} \right] Z_{\mu'\nu'} \frac{\delta}{\delta J_0(x_2)^\mu} (J_0, \Delta_Z J_0) = \\
&\left[g^{\mu'\rho} g^{\nu'\alpha} \frac{\delta^{(2)}}{\delta J_-(x)^\rho \delta J_+(x)^\alpha} \right] \left(\partial_{\mu'} \frac{\delta}{\delta J(x)^{\nu'}} - \partial_{\nu'} \frac{\delta}{\delta J(x)^{\mu'}} \right) \frac{\delta}{\delta J_0(x_2)^\mu} (J_0, \Delta_Z J_0) = \\
&\left[g^{\mu'\rho} g^{\nu'\alpha} \frac{\delta^{(2)}}{\delta J_-(x)^\rho \delta J_+(x)^\alpha} \right] \left[\partial_{\mu'} \frac{\delta}{\delta J_0(x)^{\nu'}} - \partial_{\nu'} \frac{\delta}{\delta J_0(x)^{\mu'}} \frac{\delta}{\delta J_0(x_2)^\mu} \right] (J_0, \Delta_Z J_0).
\end{aligned} \tag{B.20}$$

In the case of the field tensor terms $(W^{\mu\nu}, Z^{\mu\nu}, F^{\mu\nu})$, due to the covariant derivative of

the field terms $(\partial_\mu \Phi)$, we try the following procedure via Dirac's delta:

$$\begin{aligned} & \left[g^{\mu' \rho} g^{\nu' \alpha} \frac{\delta^{(2)}}{\delta J_- (x)^\rho \delta J_+ (x)^\alpha} \right] \int d^4 y \delta^{(4)} (x - y) \left[\frac{\partial}{\partial y^{\mu'}} \frac{\delta}{\delta J_0 (y)^{\nu'}} \frac{\delta}{\delta J_0 (x_2)^\mu} - \frac{\partial}{\partial y^{\nu'}} \frac{\delta}{\delta J_0 (y)^{\mu'}} \frac{\delta}{\delta J_0 (x_2)^\mu} \right] (J_0, \Delta_Z J_0) = \\ & \left[g^{\mu' \rho} g^{\nu' \alpha} \frac{\delta^{(2)}}{\delta J_- (x)^\rho \delta J_+ (x)^\alpha} \right] \int d^4 y \delta^{(4)} (x - y) \left[\frac{\partial}{\partial y^{\mu'}} \Delta_Z (x_2 - y)_{\mu\nu'} - \frac{\partial}{\partial y^{\nu'}} \Delta_Z (x_2 - y)_{\mu\mu'} \right]. \end{aligned} \quad (\text{B.21})$$

Now, note that

$$\begin{aligned} & \frac{\partial}{\partial y^{\mu'}} \Delta_Z (x_2 - y)_{\mu\nu'} = \int \frac{d^4 p_2}{(2\pi)^4} \frac{i}{p_2^2 - M_Z^2 + i\varepsilon} \left(-g_{\mu\nu'} + \frac{p_{2\mu} p_{2\nu'}}{M_Z^2} \right) \frac{\partial}{\partial y^{\mu'}} e^{-ip_2 \cdot (x_2 - y)} \\ & = \int \frac{d^4 p_2}{(2\pi)^4} \frac{i}{p_2^2 - M_Z^2 + i\varepsilon} \left(-g_{\nu'\mu} + \frac{p_{2\nu'} p_{2\mu}}{M_Z^2} \right) (+ip_2)_{\mu'} e^{-ip_2 \cdot (x_2 - y)} \\ & = (+ip_2)_{\mu'} \Delta_Z (x_2 - y)_{\mu\nu'}. \end{aligned} \quad (\text{B.22})$$

Thereby, we obtain

$$\begin{aligned} & \left[g^{\mu' \rho} g^{\nu' \alpha} \frac{\delta^{(2)}}{\delta J_- (x)^\rho \delta J_+ (x)^\alpha} \right] \int d^4 y \delta^{(4)} (x - y) \left[\frac{\partial}{\partial y^{\mu'}} \Delta_Z (x_2 - y)_{\mu\nu'} - \frac{\partial}{\partial y^{\nu'}} \Delta_Z (x_2 - y)_{\mu\mu'} \right] = \\ & = \left[g^{\mu' \rho} g^{\nu' \alpha} \frac{\delta^{(2)}}{\delta J_- (x)^\rho \delta J_+ (x)^\alpha} \right] \int d^4 y \delta^{(4)} (x - y) \left[(+ip_2)_{\mu'} \Delta_Z (x_2 - y)_{\mu\nu'} - (+ip_2)_{\nu'} \Delta_Z (x_2 - y)_{\mu\mu'} \right] \\ & = i \left[g^{\mu' \rho} g^{\nu' \alpha} \frac{\delta^{(2)}}{\delta J_- (x)^\rho \delta J_+ (x)^\alpha} \right] \left[(p_2)_{\mu'} \Delta_Z (x_2 - y)_{\mu\nu'} - (p_2)_{\nu'} \Delta_Z (x_2 - y)_{\mu\mu'} \right] \\ & = i g^{\mu' \rho} g^{\nu' \alpha} \frac{\delta^{(2)}}{\delta J_- (x)^\rho \delta J_+ (x)^\alpha} \left[(p_2)_{\mu'} \Delta_Z (x_2 - y)_{\mu\nu'} - (p_2)_{\nu'} \Delta_Z (x_2 - y)_{\mu\mu'} \right]. \end{aligned} \quad (\text{B.23})$$

Therefore, the Z boson contribution implies:

$$\begin{aligned} G_Z &= -i \frac{e}{s_W} \frac{c_W}{2} g^{\mu' \rho} g^{\nu' \alpha} \int d^4 x \left[\left(W_{\mu'\nu'}^+ \frac{\delta}{\delta J_+ (x)^\rho} - W_{\mu'\nu'}^- \frac{\delta}{\delta J_- (x)^\rho} \right) \Delta_Z (x_2 - x)_{\mu\alpha} \right. \\ & \quad \left. + \frac{\delta^{(2)}}{\delta J_- (x)^\rho \delta J_+ (x)^\alpha} \int d^4 y \delta^{(4)} (x - y) \left[\frac{\partial}{\partial y^{\mu'}} \Delta_Z (x_2 - y)_{\mu\nu'} - \frac{\partial}{\partial y^{\nu'}} \Delta_Z (x_2 - y)_{\mu\mu'} \right] \right] \\ &= -i \frac{e}{s_W} \frac{c_W}{2} (2) g^{\mu' \rho} g^{\nu' \alpha} \int d^4 x \left[\Delta_Z (x_2 - x)_{\mu\alpha} \left(W_{\mu'\nu'}^+ \frac{\delta}{\delta J_+ (x)^\rho} - W_{\mu'\nu'}^- \frac{\delta}{\delta J_- (x)^\rho} \right) \right. \\ & \quad \left. + i \left[(p_2)_{\mu'} \Delta_Z (x_2 - x)_{\mu\nu'} - (p_2)_{\nu'} \Delta_Z (x_2 - x)_{\mu\mu'} \right] \frac{\delta^{(2)}}{\delta J_- (x)^\rho \delta J_+ (x)^\alpha} \right]. \end{aligned} \quad (\text{B.24})$$

Now, regarding the photon contribution for the Green function, we similarly obtain the

terms as in the Z boson coupling case (B.18):

$$\begin{aligned}
G_\gamma &= ie \frac{\delta}{\delta J(x_3)^\nu} \int d^4x \left(W_{\mu'\nu'}^+ W_{\nu'}^{\mu'} A^{\nu'} - W_{\mu'\nu'}^- W_{\nu'}^{\mu'} A^{\nu'} + F_{\mu'\nu'} W_{\nu'}^{\mu'} W_{\nu'}^- \right) \frac{1}{2} (J, \Delta_\gamma J) \\
&= ieg^{\mu'\rho} g^{\nu'\alpha} \frac{\delta}{\delta J(x_3)^\nu} \int d^4x \left(W_{\mu'\nu'}^+ W_{\rho}^- A_\alpha - W_{\mu'\nu'}^- W_{\rho}^+ A_\alpha + F_{\mu'\nu'} W_{\rho}^+ W_{\alpha}^- \right) \frac{1}{2} (J, \Delta_\gamma J) \\
&= i \frac{1}{2} (2) eg^{\mu'\rho} g^{\nu'\alpha} \int d^4x \left(W_{\mu'\nu'}^+ \frac{\delta}{\delta J_-(x)^\rho} \frac{\delta^{(2)}}{\delta J(x_3)^\nu \delta J(x)^\alpha} - \right. \\
&\quad \left. W_{\mu'\nu'}^- W_{\rho}^+ \frac{\delta^{(2)}}{\delta J(x_3)^\nu \delta J(x)^\alpha} + \frac{\delta}{\delta J(x_3)^\nu} F_{\mu'\nu'} W_{\rho}^+ W_{\alpha}^- \right) (J, \Delta_\gamma J) \\
&= ieg^{\mu'\rho} g^{\nu'\alpha} \int d^4x \left[\Delta_\gamma(x_3 - x)_{\nu\alpha} \left(W_{\mu'\nu'}^+ \frac{\delta}{\delta J_+(x)^\rho} - W_{\mu'\nu'}^- \frac{\delta}{\delta J_-(x)^\rho} \right) + \right. \\
&\quad \left. i \left[(p_3)_{\mu'} \Delta_\gamma(x_3 - x)_{\nu\nu'} - (p_3)_{\nu'} \Delta_\gamma(x_3 - x)_{\nu\mu'} \right] \frac{\delta^{(2)}}{\delta J_-(x)^\rho \delta J_+(x)^\alpha} \right] \tag{B.25}
\end{aligned}$$

The (2) factor appears due to reciprocity of the photon propagator, like in the Z boson case. Now, we need to mix the contributions for the Green function with the W boson sources in order to form the loop, thereby, we mix the W boson source-derivatives in a good way:

$$\begin{aligned}
G^{(1)}(x_1, x_2, x_3) &= -i^3 \frac{1}{3!} \left(-\frac{eM_W}{s_W} iei \frac{e}{s_W} c_W \right) g^{\lambda\sigma} \int d^4x \int d^4y \int d^4z \Delta_F(x - x_1) \frac{\delta^{(2)}}{\delta J_-(x)^\lambda \delta J_+(x)^\sigma} \\
&\times g^{\mu'\rho} g^{\nu'\alpha} \left[\Delta_Z(x_2 - y)_{\mu\alpha} \left(W_{\mu'\nu'}^+ \frac{\delta}{\delta J_+(y)^\rho} - W_{\mu'\nu'}^- \frac{\delta}{\delta J_-(y)^\rho} \right) \right. \\
&\quad \left. + i \left[(p_2)_{\mu'} \Delta_Z(x_2 - y)_{\mu\nu'} - (p_2)_{\nu'} \Delta_Z(x_2 - y)_{\mu\mu'} \right] \frac{\delta^{(2)}}{\delta J_-(y)^\rho \delta J_+(y)^\alpha} \right] \\
&\times g^{\alpha'\tau} g^{\beta'\varepsilon} \left[\Delta_\gamma(x_3 - z)_{\nu\varepsilon} \left(W_{\alpha'\beta'}^+ \frac{\delta}{\delta J_+(z)^\tau} - W_{\alpha'\beta'}^- \frac{\delta}{\delta J_-(z)^\tau} \right) \right. \\
&\quad \left. + i \left[(p_3)_{\alpha'} \Delta_\gamma(x_3 - z)_{\nu\beta'} - (p_3)_{\beta'} \Delta_\gamma(x_3 - z)_{\nu\alpha'} \right] \frac{\delta^{(2)}}{\delta J_-(z)^\tau \delta J_+(z)^\varepsilon} \right] \frac{1}{3!} (J_-, \Delta_W J_+)^3. \tag{B.26}
\end{aligned}$$

We connect the internal W boson lines of the Higgs with the photon and Z boson. Concerning symmetry factors, there is a 3! factor due to the interchange of the three vertices in the diagram. And an additional 3! factor due to the symmetry of the cubic term $(J_-, \Delta_W J_+)^3$.

$$\begin{aligned}
G^{(1)}(x_1, x_2, x_3) = & -i \left(\frac{e^3 M_W c_W}{s_W^2} \right) g^{\lambda\sigma} \int d^4x \int d^4y \int d^4z \Delta_F(x - x_1) \\
& \times g^{\mu'\rho} g^{\nu'\alpha} \left[\Delta_Z(x_2 - y)_{\mu\alpha} \left(\frac{\delta}{\delta J_+(y)^\rho} \frac{\delta}{\delta J_+(x)^\sigma} W_{\mu'\nu'}^+(J_-, \Delta_W J_+) - W_{\mu'\nu'}^- \frac{\delta^{(2)}}{\delta J_-(y)^\rho \delta J_+(x)^\sigma} (J_-, \Delta_W J_+) \right) \right. \\
& + i \left[(p_2)_{\mu'} \Delta_Z(x_2 - y)_{\mu\nu'} - (p_2)_{\nu'} \Delta_Z(x_2 - y)_{\mu\mu'} \right] \frac{\delta}{\delta J_+(y)^\alpha} \frac{\delta^{(2)}(J_-, \Delta_W J_+)}{\delta J_+(x)^\sigma \delta J_-(y)^\rho} \Big] \\
& \times g^{\alpha'\tau} g^{\beta'\varepsilon} \left[\Delta_\gamma(x_3 - z)_{\nu\varepsilon} \left(W_{\alpha'\beta'}^+ \frac{\delta}{\delta J_-(x)^\lambda} \frac{\delta}{\delta J_+(z)^\tau} - \frac{\delta}{\delta J_-(x)^\lambda} W_{\alpha'\beta'}^- \frac{\delta}{\delta J_-(z)^\varepsilon} \right) (J_-, \Delta_W J_+) \right. \\
& + i \left[(p_3)_{\alpha'} \Delta_\gamma(x_3 - z)_{\nu\beta'} - (p_3)_{\beta'} \Delta_\gamma(x_3 - z)_{\nu\alpha'} \right] \frac{\delta}{\delta J_-(z)^\tau} \frac{\delta^{(2)}(J_-, \Delta_W J_+)}{\delta J_-(x)^\lambda \delta J_+(z)^\varepsilon} \Big] (J_-, \Delta_W J_+).
\end{aligned}$$

$$\begin{aligned}
G^{(1)}(x_1, x_2, x_3) = & -i \left(\frac{e^3 M_W c_W}{s_W^2} \right) g^{\lambda\sigma} \int d^4x \int d^4y \int d^4z \Delta_F(x - x_1) \\
& \times g^{\mu'\rho} g^{\nu'\alpha} \left[\Delta_Z(x_2 - y)_{\mu\alpha} \left(\frac{\delta}{\delta J_+(x)^\sigma} W_{\mu'\nu'}^+(J_-, \Delta_W J_+) \frac{\delta}{\delta J_+(y)^\rho} - W_{\mu'\nu'}^- \Delta_W(y - x)_{\rho\sigma} \right) \right. \\
& + i \left[(p_2)_{\mu'} \Delta_Z(x_2 - y)_{\mu\nu'} - (p_2)_{\nu'} \Delta_Z(x_2 - y)_{\mu\mu'} \right] \frac{\delta}{\delta J_+(y)^\alpha} \Delta_W(y - x)_{\rho\sigma} \Big] \\
& \times g^{\alpha'\tau} g^{\beta'\varepsilon} \left[\Delta_\gamma(x_3 - z)_{\nu\varepsilon} \left(\Delta_W(x - z)_{\lambda\tau} W_{\alpha'\beta'}^+ - \frac{\delta}{\delta J_-(x)^\lambda} W_{\alpha'\beta'}^- (J_-, \Delta_W J_+) \frac{\delta}{\delta J_-(z)^\varepsilon} \right) \right. \\
& + i \left[(p_3)_{\alpha'} \Delta_\gamma(x_3 - z)_{\nu\beta'} - (p_3)_{\beta'} \Delta_\gamma(x_3 - z)_{\nu\alpha'} \right] \frac{\delta}{\delta J_-(z)^\tau} \Delta_W(x - z)_{\lambda\varepsilon} \Big] (J_-, \Delta_W J_+). \tag{B.27}
\end{aligned}$$

The term of the kind $\frac{\delta}{\delta J_+(x)^\sigma} W_{\mu'\nu'}^+(J_-, \Delta_W J_+)$ is well known by the procedure of $\frac{\delta}{\delta J_0(x_2)^\mu} Z_{\mu'\nu'}(J_0, \Delta_Z J_0)$ in (B.23)

$$\begin{aligned}
G^{(1)}(x_1, x_2, x_3) = & -i \left(\frac{e^3 M_W c_W}{s_W^2} \right) g^{\lambda\sigma} \int d^4x \int d^4y \int d^4z \Delta_F(x - x_1) \\
& \times g^{\mu'\rho} g^{\nu'\alpha} \left[\Delta_Z(x_2 - y)_{\mu\alpha} \left(i \left[(p_2)_{\mu'} \Delta_W(y - x)_{\nu'\sigma} - (p_2)_{\nu'} \Delta_W(y - x)_{\mu'\sigma} \right] \frac{\delta}{\delta J_+(y)^\rho} - \Delta_W(y - x)_{\rho\sigma} W_{\mu'\nu'}^- \right) \right. \\
& + i \left[(p_2)_{\mu'} \Delta_Z(x_2 - y)_{\mu\nu'} - (p_2)_{\nu'} \Delta_Z(x_2 - y)_{\mu\mu'} \right] \Delta_W(y - x)_{\rho\sigma} \frac{\delta}{\delta J_+(y)^\alpha} \Big] \\
& \times g^{\alpha'\tau} g^{\beta'\varepsilon} \left[\Delta_\gamma(x_3 - z)_{\nu\varepsilon} \left(\Delta_W(x - z)_{\lambda\tau} W_{\alpha'\beta'}^+ - i \left[(p_3)_{\alpha'} \Delta_W(x - z)_{\lambda\beta'} - (p_3)_{\beta'} \Delta_W(x - z)_{\lambda\alpha'} \right] \frac{\delta}{\delta J_-(z)^\varepsilon} \right) \right. \\
& + i \left[(p_3)_{\alpha'} \Delta_\gamma(x_3 - z)_{\nu\beta'} - (p_3)_{\beta'} \Delta_\gamma(x_3 - z)_{\nu\alpha'} \right] \Delta_W(x - z)_{\lambda\varepsilon} \frac{\delta}{\delta J_-(z)^\tau} \Big] (J_-, \Delta_W J_+).
\end{aligned}$$

Define a new tensor structure in order to organize the terms:

$$\mathcal{F}(p, x - y, V)_{\alpha\beta\gamma} = i \left[(p)_\alpha \Delta_V(x - y)_{\beta\gamma} - (p)_\gamma \Delta_V(x - y)_{\beta\alpha} \right], \tag{B.28}$$

and we therefore obtain

$$\begin{aligned}
G^{(1)}(x_1, x_2, x_3) = & -i \left(\frac{e^3 M_W c_W}{s_W^2} \right) g^{\lambda\sigma} g^{\mu'\rho} g^{\nu'\alpha} g^{\alpha'\tau} g^{\beta'\varepsilon} \int d^4 x d^4 y d^4 z \Delta_F(x - x_1) \left[\Delta_Z(x_2 - y)_{\mu\alpha} \right. \\
& \times \left(\mathcal{F}(\ell, y - x, W)_{\mu'\sigma\nu'} \frac{\delta}{\delta J_+(y)^\rho} - \Delta_W(y - x)_{\rho\sigma} W_{\mu'\nu'}^- \right) + \mathcal{F}(p_2, x_2 - y, Z)_{\mu'\mu\nu'} \Delta_W(y - x)_{\rho\sigma} \frac{\delta}{\delta J_+(y)^\alpha} \Big] \\
& \times \left[\Delta_\gamma(x_3 - z)_{\nu\varepsilon} \left(\Delta_W(x - z)_{\lambda\tau} W_{\alpha'\beta'}^+ - \mathcal{F}(\ell_3, x - z, W)_{\alpha'\lambda\beta'} \frac{\delta}{\delta J_-(z)^\varepsilon} \right) \right. \\
& \left. + \mathcal{F}(p_3, x_3 - z, \gamma)_{\alpha'\nu\beta'} \Delta_W(x - z)_{\lambda\varepsilon} \frac{\delta}{\delta J_-(z)^\tau} \right] (J_-, \Delta_W J_+). \tag{B.29}
\end{aligned}$$

Expanding,

$$\begin{aligned}
G^{(1)}(x_1, x_2, x_3) = & -i \left(\frac{e^3 M_W c_W}{s_W^2} \right) g^{\lambda\sigma} g^{\mu'\rho} g^{\nu'\alpha} g^{\alpha'\tau} g^{\beta'\varepsilon} \int d^4 x \int d^4 y \int d^4 z \Delta_F(x - x_1) \\
& \times \left[\Delta_Z(x_2 - y)_{\mu\alpha} \Delta_\gamma(x_3 - z)_{\nu\varepsilon} \Delta_W(x - z)_{\lambda\tau} \mathcal{F}(\ell, y - x, W)_{\mu'\sigma\nu'} W_{\alpha'\beta'}^+(z) \frac{\delta}{\delta J_+(y)^\rho} (J_-, \Delta_W J_+) \right. \\
& - \Delta_Z(x_2 - y)_{\mu\alpha} \Delta_\gamma(x_3 - z)_{\nu\varepsilon} \mathcal{F}(\ell, y - x, W)_{\mu'\sigma\nu'} \mathcal{F}(\ell_3, x - z, W)_{\alpha'\lambda\beta'} \frac{\delta}{\delta J_-(z)^\varepsilon} \frac{\delta}{\delta J_+(y)^\rho} (J_-, \Delta_W J_+) \\
& + \Delta_Z(x_2 - y)_{\mu\alpha} \Delta_W(x - z)_{\lambda\varepsilon} \mathcal{F}(\ell, y - x, W)_{\mu'\sigma\nu'} \mathcal{F}(p_3, x_3 - z, \gamma)_{\alpha'\nu\beta'} \frac{\delta}{\delta J_-(z)^\tau} \frac{\delta}{\delta J_+(y)^\rho} (J_-, \Delta_W J_+) \\
& - \Delta_Z(x_2 - y)_{\mu\alpha} \Delta_W(y - x)_{\rho\sigma} \Delta_\gamma(x_3 - z)_{\nu\varepsilon} \Delta_W(x - z)_{\lambda\tau} W_{\alpha'\beta'}^+(z) W_{\mu'\nu'}^-(y) (J_-, \Delta_W J_+) \\
& + \Delta_Z(x_2 - y)_{\mu\alpha} \Delta_W(y - x)_{\rho\sigma} \Delta_\gamma(x_3 - z)_{\nu\varepsilon} \mathcal{F}(\ell_3, x - z, W)_{\alpha'\lambda\beta'} \frac{\delta}{\delta J_-(z)^\varepsilon} W_{\mu'\nu'}^-(y) (J_-, \Delta_W J_+) \\
& - \Delta_Z(x_2 - y)_{\mu\alpha} \Delta_W(y - x)_{\rho\sigma} \Delta_W(x - z)_{\lambda\varepsilon} \mathcal{F}(p_3, x_3 - z, \gamma)_{\alpha'\nu\beta'} \frac{\delta}{\delta J_-(z)^\tau} W_{\mu'\nu'}^-(y) (J_-, \Delta_W J_+) \\
& + \Delta_W(y - x)_{\rho\sigma} \Delta_\gamma(x_3 - z)_{\nu\varepsilon} \Delta_W(x - z)_{\lambda\tau} \mathcal{F}(p_2, x_2 - y, Z)_{\mu'\mu\nu'} W_{\alpha'\beta'}^+(z) \frac{\delta}{\delta J_+(y)^\alpha} (J_-, \Delta_W J_+) \\
& - \Delta_W(y - x)_{\rho\sigma} \Delta_\gamma(x_3 - z)_{\nu\varepsilon} \mathcal{F}(p_2, x_2 - y, Z)_{\mu'\mu\nu'} \mathcal{F}(\ell_3, x - z, W)_{\alpha'\lambda\beta'} \frac{\delta}{\delta J_-(z)^\varepsilon} \frac{\delta}{\delta J_+(y)^\alpha} (J_-, \Delta_W J_+) \\
& \left. + \Delta_W(x - z)_{\lambda\varepsilon} \Delta_W(y - x)_{\rho\sigma} \mathcal{F}(p_2, x_2 - y, Z)_{\mu'\mu\nu'} \mathcal{F}(\ell_3, x_3 - z, \gamma)_{\alpha'\nu\beta'} \frac{\delta}{\delta J_-(z)^\tau} \frac{\delta}{\delta J_+(y)^\alpha} (J_-, \Delta_W J_+) \right] \tag{B.30}
\end{aligned}$$

$$\begin{aligned}
G^{(1)}(x_1, x_2, x_3) = & -i \left(\frac{e^3 M_W c_W}{s_W^2} \right) g^{\lambda\sigma} g^{\mu'\rho} g^{\nu'\alpha} g^{\alpha'\tau} g^{\beta'\varepsilon} \int d^4 x \int d^4 y \int d^4 z \Delta_F(x - x_1) \\
& \times \left[\Delta_Z(x_2 - y)_{\mu\alpha} \Delta_\gamma(x_3 - z)_{\nu\varepsilon} \Delta_W(x - z)_{\lambda\tau} \mathcal{F}(\ell, y - x, W)_{\mu'\sigma\nu'} \mathcal{F}(\ell_2, z - y, W)_{\alpha'\rho\beta'} \right. \\
& - \Delta_Z(x_2 - y)_{\mu\alpha} \Delta_\gamma(x_3 - z)_{\nu\varepsilon} \Delta_W(z - y)_{\varepsilon\rho} \mathcal{F}(\ell, y - x, W)_{\mu'\sigma\nu'} \mathcal{F}(\ell_3, x - z, W)_{\alpha'\lambda\beta'} \\
& + \Delta_Z(x_2 - y)_{\mu\alpha} \Delta_W(x - z)_{\lambda\varepsilon} \Delta_W(z - y)_{\tau\rho} \mathcal{F}(\ell, y - x, W)_{\mu'\sigma\nu'} \mathcal{F}(p_3, x_3 - z, \gamma)_{\alpha'\nu\beta'} \\
& - \Delta_Z(x_2 - y)_{\mu\alpha} \Delta_\gamma(x_3 - z)_{\nu\varepsilon} \Delta_W(y - x)_{\rho\sigma} \Delta_W(x - z)_{\lambda\tau} W_{\alpha'\beta'}^+(z) W_{\mu'\nu'}^-(y) (J_-, \Delta_W J_+) \\
& + \Delta_Z(x_2 - y)_{\mu\alpha} \Delta_\gamma(x_3 - z)_{\nu\varepsilon} \Delta_W(y - x)_{\rho\sigma} \mathcal{F}(\ell_3, x - z, W)_{\alpha'\lambda\beta'} \mathcal{F}(\ell_2, z - y, W)_{\mu'\varepsilon\nu'} \\
& - \Delta_Z(x_2 - y)_{\mu\alpha} \Delta_W(y - x)_{\rho\sigma} \Delta_W(x - z)_{\lambda\varepsilon} \mathcal{F}(p_3, x_3 - z, \gamma)_{\alpha'\nu\beta'} \mathcal{F}(\ell_2, z - y, W)_{\mu'\tau\nu'} \\
& + \Delta_\gamma(x_3 - z)_{\nu\varepsilon} \Delta_W(x - z)_{\lambda\tau} \Delta_W(y - x)_{\rho\sigma} \mathcal{F}(p_2, x_2 - y, Z)_{\mu'\mu\nu'} \mathcal{F}(\ell_2, z - y, W)_{\alpha'\alpha\beta'} \\
& - \Delta_\gamma(x_3 - z)_{\nu\varepsilon} \Delta_W(z - y)_{\varepsilon\alpha} \Delta_W(y - x)_{\rho\sigma} \mathcal{F}(p_2, x_2 - y, Z)_{\mu'\mu\nu'} \mathcal{F}(\ell_3, x - z, W)_{\alpha'\lambda\beta'} \\
& \left. + \Delta_W(x - z)_{\lambda\varepsilon} \Delta_W(y - x)_{\rho\sigma} \Delta_W(z - y)_{\tau\alpha} \mathcal{F}(p_2, x_2 - y, Z)_{\mu'\mu\nu'} \mathcal{F}(\ell_3, x_3 - z, \gamma)_{\alpha'\nu\beta'} \right]. \tag{B.31}
\end{aligned}$$

Now, consider

$$\begin{aligned}
& W_{\alpha'\beta'}^+(z) W_{\mu'\nu'}^-(y) (J_-, \Delta_W J_+) = \\
& = \left(\partial_{\alpha'} \frac{\delta}{\delta J_-(z)^{\beta'}} - \partial_{\beta'} \frac{\delta}{\delta J_-(z)^{\alpha'}} \right) \left(\partial_{\mu'} \frac{\delta}{\delta J_+(y)^{\nu'}} - \partial_{\nu'} \frac{\delta}{\delta J_+(y)^{\mu'}} \right) (J_-, \Delta_W J_+) \\
& = \left(\partial_{\alpha'} \frac{\delta}{\delta J_-(z)^{\beta'}} \partial_{\mu'} \frac{\delta}{\delta J_+(y)^{\nu'}} - \partial_{\alpha'} \frac{\delta}{\delta J_-(z)^{\beta'}} \partial_{\nu'} \frac{\delta}{\delta J_+(y)^{\mu'}} \right. \\
& \quad \left. - \partial_{\beta'} \frac{\delta}{\delta J_-(z)^{\alpha'}} \partial_{\mu'} \frac{\delta}{\delta J_+(y)^{\nu'}} + \partial_{\beta'} \frac{\delta}{\delta J_-(z)^{\alpha'}} \partial_{\nu'} \frac{\delta}{\delta J_+(y)^{\mu'}} \right) (J_-, \Delta_W J_+) \\
& = i^2 (-\ell_2)_{\alpha'} (\ell_2)_{\mu'} \Delta_W (z-y)_{\beta'\nu'} - i^2 (-\ell_2)_{\alpha'} (\ell_2)_{\nu'} \Delta_W (z-y)_{\beta'\mu'} \\
& \quad - i^2 (-\ell_2)_{\beta'} (\ell_2)_{\mu'} \Delta_W (z-y)_{\alpha'\nu'} + i^2 (-\ell_2)_{\beta'} (\ell_2)_{\nu'} \Delta_W (z-y)_{\alpha'\mu'} \\
& = (\ell_2)_{\alpha'} (\ell_2)_{\mu'} \Delta_W (z-y)_{\beta'\nu'} - (\ell_2)_{\alpha'} (\ell_2)_{\nu'} \Delta_W (z-y)_{\beta'\mu'} \\
& \quad - (\ell_2)_{\beta'} (\ell_2)_{\mu'} \Delta_W (z-y)_{\alpha'\nu'} + (\ell_2)_{\beta'} (\ell_2)_{\nu'} \Delta_W (z-y)_{\alpha'\mu'} \\
& = (\ell_2)_{\alpha'} \mathcal{F}(\ell_2, z-y, W)_{\mu'\beta'\nu'} + (\ell_2)_{\beta'} \mathcal{F}(\ell_2, z-y, W)_{\nu'\alpha'\mu'}, \tag{B.32}
\end{aligned}$$

thereby, the Green function reads

$$\begin{aligned}
G^{(1)}(x_1, x_2, x_3) = & -i \left(\frac{e^3 M_W c_W}{s_W^2} \right) g^{\lambda\sigma} g^{\mu'\rho} g^{\nu'\alpha} g^{\alpha'\tau} g^{\beta'\varepsilon} \int d^4x \int d^4y \int d^4z \Delta_F(x-x_1) \\
& \times \left[\Delta_Z(x_2-y)_{\mu\alpha} \Delta_\gamma(x_3-z)_{\nu\varepsilon} \Delta_W(x-z)_{\lambda\tau} \mathcal{F}(\ell, y-x, W)_{\mu'\sigma\nu'} \mathcal{F}(\ell_2, z-y, W)_{\alpha'\rho\beta'} \right. \\
& - \Delta_Z(x_2-y)_{\mu\alpha} \Delta_\gamma(x_3-z)_{\nu\varepsilon} \Delta_W(z-y)_{\varepsilon\rho} \mathcal{F}(\ell, y-x, W)_{\mu'\sigma\nu'} \mathcal{F}(\ell_3, x-z, W)_{\alpha'\lambda\beta'} \\
& + \Delta_Z(x_2-y)_{\mu\alpha} \mathcal{F}(p_3, x_3-z, \gamma)_{\alpha'\nu\beta'} \Delta_W(x-z)_{\lambda\varepsilon} \Delta_W(z-y)_{\tau\rho} \mathcal{F}(\ell, y-x, W)_{\mu'\sigma\nu'} \\
& - \Delta_Z(x_2-y)_{\mu\alpha} \Delta_\gamma(x_3-z)_{\nu\varepsilon} \Delta_W(y-x)_{\rho\sigma} \Delta_W(x-z)_{\lambda\tau} (\ell_2)_{\alpha'} \mathcal{F}(\ell_2, z-y, W)_{\mu'\beta'\nu'} \\
& - \Delta_Z(x_2-y)_{\mu\alpha} \Delta_\gamma(x_3-z)_{\nu\varepsilon} \Delta_W(y-x)_{\rho\sigma} \Delta_W(x-z)_{\lambda\tau} (\ell_2)_{\beta'} \mathcal{F}(\ell_2, z-y, W)_{\nu'\alpha'\mu'} \\
& + \Delta_Z(x_2-y)_{\mu\alpha} \Delta_\gamma(x_3-z)_{\nu\varepsilon} \Delta_W(y-x)_{\rho\sigma} \mathcal{F}(\ell_3, x-z, W)_{\alpha'\lambda\beta'} \mathcal{F}(\ell_2, z-y, W)_{\mu'\varepsilon\nu'} \\
& - \Delta_Z(x_2-y)_{\mu\alpha} \mathcal{F}(p_3, x_3-z, \gamma)_{\alpha'\nu\beta'} \Delta_W(y-x)_{\rho\sigma} \Delta_W(x-z)_{\lambda\varepsilon} \mathcal{F}(\ell_2, z-y, W)_{\mu'\tau\nu'} \\
& + \Delta_\gamma(x_3-z)_{\nu\varepsilon} \mathcal{F}(p_2, x_2-y, Z)_{\mu'\mu\nu'} \Delta_W(x-z)_{\lambda\tau} \Delta_W(y-x)_{\rho\sigma} \mathcal{F}(\ell_2, z-y, W)_{\alpha'\alpha\beta'} \\
& - \Delta_\gamma(x_3-z)_{\nu\varepsilon} \mathcal{F}(p_2, x_2-y, Z)_{\mu'\mu\nu'} \Delta_W(z-y)_{\varepsilon\alpha} \Delta_W(y-x)_{\rho\sigma} \mathcal{F}(\ell_3, x-z, W)_{\alpha'\lambda\beta'} \\
& \left. + \mathcal{F}(p_2, x_2-y, Z)_{\mu'\mu\nu'} \mathcal{F}(\ell_3, x_3-z, \gamma)_{\alpha'\nu\beta'} \Delta_W(x-z)_{\lambda\varepsilon} \Delta_W(y-x)_{\rho\sigma} \Delta_W(z-y)_{\tau\alpha} \right]. \tag{B.33}
\end{aligned}$$

The expression above gives 36 terms. In order to recognize the whole Green function and deduce a reduced formula, we consider the first term, which gives 4 terms in momentum space

$$\Delta_F(x-x_1) \Delta_Z(x_2-y)_{\mu\alpha} \Delta_\gamma(x_3-z)_{\nu\varepsilon} \Delta_W(x-z)_{\lambda\tau} \mathcal{F}(\ell, y-x, W)_{\mu'\sigma\nu'} \mathcal{F}(\ell_2, z-y, W)_{\alpha'\rho\beta'}, \tag{B.34}$$

thereby,

$$\begin{aligned}
& g^{\lambda\sigma} g^{\mu'\rho} g^{\nu'\alpha} g^{\alpha'\tau} g^{\beta'\varepsilon} \int d^4x d^4y d^4z \Delta_F(x-x_1) \Delta_Z(x_2-y)_{\mu\alpha} \Delta_\gamma(x_3-z)_{\nu\varepsilon} \\
& \times \Delta_W(y-x)_{\sigma\nu'} \Delta_W(z-y)_{\rho\beta'} \Delta_W(x-z)_{\lambda\tau} i\ell_{\mu'} i(\ell_2)_{\alpha'} = \\
& i^2 g^{\lambda\sigma} g^{\mu'\rho} g^{\nu'\alpha} g^{\alpha'\tau} g^{\beta'\varepsilon} \int d^4x d^4y d^4z \frac{d^4p_1 d^4p_2 d^4p_3 d^4\ell d^4\ell_2 d^4\ell_3}{(2\pi)^{24}} \frac{i}{p_1^2 - M_H^2} e^{-ip_1 \cdot (x-x_1)} \frac{-ig_{\mu\alpha}}{p_2^2 - M_Z^2} e^{-ip_2 \cdot (x_2-y)} \\
& \times \frac{-ig_{\nu\varepsilon}}{p_3^2} e^{-ip_3 \cdot (x_3-z)} \ell_{\mu'} (\ell_2)_{\alpha'} \frac{-ig_{\sigma\nu'}}{\ell^2 - M_W^2} e^{-i\ell \cdot (y-x)} \frac{-ig_{\lambda\tau}}{\ell_3^2 - M_W^2} e^{-i\ell_3 \cdot (x-z)} \frac{-ig_{\rho\beta'}}{\ell_2^2 - M_W^2} e^{-i\ell_2 \cdot (z-y)} = \\
& i^2 g^{\lambda\sigma} g^{\mu'\rho} g^{\nu'\alpha} g^{\alpha'\tau} g^{\beta'\varepsilon} \int d^4x d^4y d^4z \frac{d^4p_1 d^4p_2 d^4p_3 d^4\ell d^4\ell_2 d^4\ell_3}{(2\pi)^{24}} \frac{i}{p_1^2 - M_H^2} \frac{-ig_{\mu\alpha}}{p_2^2 - M_Z^2} \\
& \times \frac{-ig_{\nu\varepsilon}}{p_3^2} \frac{-ig_{\sigma\nu'}}{\ell^2 - M_W^2} \frac{-ig_{\rho\beta'}}{\ell_2^2 - M_W^2} \frac{-ig_{\lambda\tau}}{\ell_3^2 - M_W^2} \\
& \times e^{-ip_1 \cdot (x-x_1)} e^{-ip_2 \cdot (x_2-y)} e^{-ip_3 \cdot (x_3-z)} \ell_{\mu'} (\ell_2)_{\alpha'} e^{-i\ell \cdot (y-x)} e^{-i\ell_3 \cdot (x-z)} e^{-i\ell_2 \cdot (z-y)} = \\
& i^2 g^{\lambda\sigma} g^{\mu'\rho} g^{\nu'\alpha} g^{\alpha'\tau} g^{\beta'\varepsilon} \int d^4x d^4y d^4z \frac{d^4p_1 d^4p_2 d^4p_3 d^4\ell d^4\ell_2 d^4\ell_3}{(2\pi)^{24}} \frac{i}{p_1^2 - M_H^2} \\
& \times \frac{-ig_{\mu\alpha}}{p_2^2 - M_Z^2} \frac{-ig_{\nu\varepsilon}}{p_3^2} \frac{-ig_{\sigma\nu'}}{\ell^2 - M_W^2} \frac{-ig_{\rho\beta'}}{\ell_2^2 - M_W^2} \frac{-ig_{\lambda\tau}}{\ell_3^2 - M_W^2} \\
& \times e^{-ix \cdot (p_1 - \ell + \ell_3)} e^{-iy \cdot (-p_2 - \ell_2 + \ell)} e^{-iz \cdot (-p_3 - \ell_3 + \ell_2)} e^{ip_1 \cdot x_1} e^{-ip_2 \cdot x_2} e^{-ip_3 \cdot x_3} \ell_{\mu'} (\ell_2)_{\alpha'} = \\
& i^2 g^{\lambda\sigma} g^{\mu'\rho} g^{\nu'\alpha} g^{\alpha'\tau} g^{\beta'\varepsilon} \int \frac{d^4p_1 d^4p_2 d^4p_3 d^4\ell d^4\ell_2 d^4\ell_3}{(2\pi)^{24}} \frac{i}{p_1^2 - M_H^2} \frac{-ig_{\mu\alpha}}{p_2^2 - M_Z^2} \frac{-ig_{\nu\varepsilon}}{p_3^2} \frac{-ig_{\sigma\nu'}}{\ell^2 - M_W^2} \frac{-ig_{\rho\beta'}}{\ell_2^2 - M_W^2} \frac{-ig_{\lambda\tau}}{\ell_3^2 - M_W^2} \\
& \times \delta^{(4)}(p_1 - \ell + \ell_3) \delta^{(4)}(-p_2 - \ell_2 + \ell) \delta^{(4)}(-p_3 - \ell_3 + \ell_2) e^{ip_1 \cdot x_1} e^{-ip_2 \cdot x_2} e^{-ip_3 \cdot x_3} \ell_{\mu'} (\ell_2)_{\alpha'} = \\
& = i^2 g^{\lambda\sigma} g^{\mu'\rho} g^{\nu'\alpha} g^{\alpha'\tau} g^{\beta'\varepsilon} \int \frac{d^4p_1}{(2\pi)^4} \frac{d^4p_2}{(2\pi)^4} \frac{d^4p_3}{(2\pi)^4} \int \frac{d^4\ell}{(2\pi)^4} \left\{ \frac{i}{(p_2 + p_3)^2 - M_H^2} e^{ip_1 \cdot x_1} \frac{-ig_{\mu\alpha}}{p_2^2 - M_Z^2} e^{-ip_2 \cdot x_2} \frac{-ig_{\nu\varepsilon}}{p_3^2} e^{-ip_3 \cdot x_3} \right\} \\
& \times \delta^{(4)}(p_1 - p_2 - p_3) \frac{-ig_{\sigma\nu'}}{\ell^2 - M_W^2} \frac{-ig_{\rho\beta'}}{(\ell - p_2)^2 - M_W^2} \frac{-ig_{\lambda\tau}}{(\ell - p_2 - p_3)^2 - M_W^2} \ell_{\mu'} (\ell_2)_{\alpha'}. \tag{B.35}
\end{aligned}$$

The Dirac's deltas impose three momentum conditions $\ell_2 = \ell - p_2$; $\ell_3 = \ell_2 - p_3 \rightarrow \ell_3 = \ell - p_2 - p_3$; $p_1 = \ell - \ell_3 \rightarrow p_1 = p_2 + p_3$. Define $D_0 = \ell^2 - M_W^2$; $D_1 = (\ell - p_2)^2 - M_W^2$; $D_2 = (\ell - p_2 - p_3)^2 - M_W^2$, and the integral reads

$$\begin{aligned}
& -g^{\lambda\sigma} g^{\mu'\rho} g^{\nu'\alpha} g^{\alpha'\tau} g^{\beta'\varepsilon} \int \frac{d^4p_1}{(2\pi)^4} \frac{d^4p_2}{(2\pi)^4} \frac{d^4p_3}{(2\pi)^4} \frac{d^4\ell}{(2\pi)^4} \delta^{(4)}(p_1 - p_2 - p_3) \\
& \left\{ \frac{ie^{ip_1 \cdot x_1}}{(p_2 + p_3)^2 - M_H^2} \frac{-ig_{\mu\alpha}}{p_2^2 - M_Z^2} e^{-ip_2 \cdot x_2} \frac{-ig_{\nu\varepsilon}}{p_3^2} e^{-ip_3 \cdot x_3} \right\} \frac{-ig_{\sigma\nu'} (-ig_{\rho\beta'}) (-ig_{\lambda\tau})}{D_0 D_1 D_2} \ell_{\mu'} (\ell_2)_{\alpha'} = \\
& -g^{\lambda\sigma} g^{\mu'\rho} g^{\nu'\alpha} g^{\alpha'\tau} g^{\beta'\varepsilon} \int \frac{d^4p_1}{(2\pi)^4} \frac{d^4p_2}{(2\pi)^4} \frac{d^4p_3}{(2\pi)^4} \frac{d^4\ell}{(2\pi)^4} \delta^{(4)}(p_1 - p_2 - p_3) \\
& \times \left\{ \frac{i}{(p_2 + p_3)^2 - M_H^2} e^{ip_1 \cdot x_1} \frac{-ig_{\mu\alpha}}{p_2^2 - M_Z^2} e^{-ip_2 \cdot x_2} \frac{-ig_{\nu\varepsilon}}{p_3^2} e^{-ip_3 \cdot x_3} \right\} \frac{i}{D_0 D_1 D_2} g_{\sigma\nu'} g_{\rho\beta'} g_{\lambda\tau} \ell_{\mu'} (\ell_2)_{\alpha'}. \tag{B.36}
\end{aligned}$$

We are going to take a look on the tensor objects in the term:

$$\begin{aligned}
& g^{\lambda\sigma} g^{\mu'\rho} g^{\nu'\alpha} g^{\alpha'\tau} g^{\beta'\varepsilon} g_{\mu\alpha} g_{\nu\varepsilon} g_{\sigma\nu'} g_{\rho\beta'} \ell_{\mu'} (\ell_2)_{\alpha'} g_{\lambda\tau} = \\
& g^{\lambda\sigma} g_{\sigma\nu'} g^{\nu'\alpha} g_{\mu\alpha} g^{\mu'\rho} \ell_{\mu'} g_{\rho\beta'} g^{\beta'\varepsilon} g_{\nu\varepsilon} g^{\alpha'\tau} (\ell_2)_{\alpha'} g_{\lambda\tau} = \\
& g^{\lambda\sigma} g_{\sigma\nu'} g^{\nu'\alpha} g_{\mu\alpha} \ell_{\mu'} \delta_{\beta'}^{\mu'} g^{\beta'\varepsilon} g_{\nu\varepsilon} g^{\alpha'\tau} (\ell_2)_{\alpha'} g_{\lambda\tau} = \\
& g^{\lambda\sigma} \delta_{\alpha}^{\sigma} g_{\mu\alpha} \ell_{\mu'} g^{\mu'\varepsilon} g_{\nu\varepsilon} g^{\alpha'\tau} (\ell_2)_{\alpha'} g_{\lambda\tau} = \\
& g^{\lambda\sigma} (g_{\mu\sigma} \ell_{\mu'}) g^{\mu'\varepsilon} (g_{\nu\varepsilon} (\ell_2)_{\alpha'}) g^{\alpha'\tau} g_{\lambda\tau} = \\
& g_{\lambda\tau} g^{\lambda\sigma} (g_{\mu\sigma} \ell_{\mu'}) g^{\mu'\varepsilon} (g_{\nu\varepsilon} (\ell_2)_{\alpha'}) g^{\alpha'\tau}
\end{aligned} \tag{B.37}$$

$$\begin{aligned}
& \therefore - \int \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_2}{(2\pi)^4} \frac{d^4 p_3}{(2\pi)^4} \frac{d^4 \ell}{(2\pi)^4} \left\{ \frac{i}{(p_2 + p_3)^2 - M_H^2} e^{ip_1 \cdot x_1} \frac{-i}{p_2^2 - M_Z^2} e^{-ip_2 \cdot x_2} \frac{-i}{p_3^2} e^{-ip_3 \cdot x_3} \right\} \\
& \times \delta^{(4)}(p_1 - p_2 - p_3) \frac{i}{D_0 D_1 D_2} g_{\lambda\tau} g^{\lambda\sigma} (g_{\mu\sigma} \ell_{\mu'}) g^{\mu'\varepsilon} (g_{\nu\varepsilon} (\ell_2)_{\alpha'}) g^{\alpha'\tau}
\end{aligned} \tag{B.38}$$

This is one of the 36 terms conforming the Green function of the triangle loop contribution. And the three external propagators will be amputated by LSZ reduction formula.

B.3 LSZ Reduction Formula

With the LSZ reduction formula for our case, we are going to try the term we have been studying (B.38)

$$\begin{aligned}
& i^{1+2} \int d^4 x_1 e^{-ip_1 x_1} (\partial_1^2 + M_H^2) \int d^4 x_2 \varepsilon_{\lambda_2}^{\mu*}(p_2) e^{+ip_2 x_2} (\partial_2^2 + M_Z^2) \int d^4 x_3 \varepsilon_{\lambda_3}^{\nu*}(p_3) e^{+ip_3 x_3} (\partial_3^2) \\
& \times \left(-i \frac{e^3 M_W c_W}{s_W^2} \right) i^2 \int \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_2}{(2\pi)^4} \frac{d^4 p_3}{(2\pi)^4} \delta^{(4)}(p_1 - p_2 - p_3) \frac{d^4 \ell}{(2\pi)^4} \\
& \times \left\{ \frac{i}{(p_2 + p_3)^2 - M_H^2} e^{ip_1 \cdot x_1} \frac{-i}{p_2^2 - M_Z^2} e^{-ip_2 \cdot x_2} \frac{-i}{p_3^2} e^{-ip_3 \cdot x_3} \right\} \frac{i}{D_0 D_1 D_2} g_{\lambda\tau} g^{\lambda\sigma} (g_{\mu\sigma} \ell_{\mu'}) g^{\mu'\varepsilon} (g_{\nu\varepsilon} (\ell_2)_{\alpha'}) g^{\alpha'\tau} =
\end{aligned}$$

$$\begin{aligned}
& i^3 \int d^4 x_1 e^{-ip_1 x_1} \int d^4 x_2 \varepsilon_{\lambda_2}^{\mu*}(p_2) e^{+ip_2 x_2} \int d^4 x_3 \varepsilon_{\lambda_3}^{\nu*}(p_3) e^{+ip_3 x_3} \\
& \times \left(-i \frac{e^3 M_W c_W}{s_W^2} \right) \int \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_2}{(2\pi)^4} \frac{d^4 p_3}{(2\pi)^4} \frac{d^4 \ell}{(2\pi)^4} \delta^{(4)}(p_1 - p_2 - p_3) \frac{i}{D_0 D_1 D_2} \\
& \times \left\{ \frac{i}{(p_2 + p_3)^2 - M_H^2} (-p_1^2 + M_H^2) e^{ip_1 \cdot x_1} \frac{-i}{p_2^2 - M_Z^2} (-p_2^2 + M_Z^2) e^{-ip_2 \cdot x_2} \frac{-i}{p_3^2} (-p_3^2) e^{-ip_3 \cdot x_3} \right\} \\
& \times g_{\lambda\tau} g^{\lambda\sigma} (g_{\mu\sigma} \ell_{\mu'}) g^{\mu'\varepsilon} (g_{\nu\varepsilon} (\ell_2)_{\alpha'}) g^{\alpha'\tau} =
\end{aligned}$$

$$\begin{aligned}
& i^{1+2} \varepsilon_{\lambda_2}^{\mu*}(p_2) \varepsilon_{\lambda_3}^{\nu*}(p_3) \int d^4 x_1 e^{-i p_1 x_1} \int d^4 x_2 e^{+i p_2 x_2} \int d^4 x_3 e^{+i p_3 x_3} \\
& \times \left(-i \frac{e^3 M_W c_W}{s_W^2} \right) (i)^3 (i)(-1) \int \frac{d^4 p_1}{(2\pi)^4} \frac{d^4 p_2}{(2\pi)^4} \frac{d^4 p_3}{(2\pi)^4} \delta^{(4)}(p_1 - p_2 - p_3) \{ e^{i p_1 \cdot x_1} e^{-i p_2 \cdot x_2} e^{-i p_3 \cdot x_3} \} \\
& \times \int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{D_0 D_1 D_2} g_{\lambda\tau} g^{\lambda\sigma} (g_{\mu\sigma} \ell_{\mu'}) g^{\mu'\varepsilon} (g_{\nu\varepsilon} (\ell_2)_{\alpha'}) g^{\alpha'\tau} = \\
& i \varepsilon_{\lambda_2}^{\mu*}(p_2) \varepsilon_{\lambda_3}^{\nu*}(p_3) \int d^4 x_1 e^{-i p_1 x_1} \int d^4 x_2 e^{+i p_2 x_2} \int d^4 x_3 e^{+i p_3 x_3} \delta^{(4)}(x_2 - x_1) \delta^{(4)}(x_3 - x_1) \\
& \times \left(-i \frac{e^3 M_W c_W}{s_W^2} \right) \int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{D_0 D_1 D_2} g_{\lambda\tau} g^{\lambda\sigma} (g_{\mu\sigma} \ell_{\mu'}) g^{\mu'\varepsilon} (g_{\nu\varepsilon} (\ell_2)_{\alpha'}) g^{\alpha'\tau} = \\
& i \varepsilon_{\lambda_2}^{\mu*}(p_2) \varepsilon_{\lambda_3}^{\nu*}(p_3) \int d^4 x_1 e^{-i(p_1 - p_2 - p_3) \cdot x_1} \\
& \times \left(-i \frac{e^3 M_W c_W}{s_W^2} \right) \int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{D_0 D_1 D_2} g_{\lambda\tau} g^{\lambda\sigma} (g_{\mu\sigma} \ell_{\mu'}) g^{\mu'\varepsilon} (g_{\nu\varepsilon} (\ell_2)_{\alpha'}) g^{\alpha'\tau} = \\
& (2\pi)^4 \delta^{(4)}(p_1 - p_2 - p_3) \left(\frac{e^3 M_W c_W}{s_W^2} \right) \varepsilon_{\lambda_2}^{\mu*}(p_2) \varepsilon_{\lambda_3}^{\nu*}(p_3) \\
& \times \int \frac{d^4 \ell}{(2\pi)^4} \frac{1}{D_0 D_1 D_2} g_{\lambda\tau} g^{\lambda\sigma} (g_{\mu\sigma} \ell_{\mu'}) g^{\mu'\varepsilon} (g_{\nu\varepsilon} (\ell_2)_{\alpha'}) g^{\alpha'\tau} \tag{B.39}
\end{aligned}$$

As a consequence of above, the on-shell amplitude of the decay process has been obtained by amputating off-shell propagators which yield to on-shell external lines. The expression above is a term of the S-Matrix element, so we compare with the expression (2.14) in order to obtain $i\mathcal{M}$. By means of the antisymmetric tensor defined (2.26) satisfying the relations:

$$\mathcal{V}_{\mu\nu\rho}(p_1, p_2, p_3) = -\mathcal{V}_{\mu\rho\nu}(p_1, p_3, p_2) = \mathcal{V}_{\rho\mu\nu}(p_3, p_1, p_2) \tag{B.40}$$

With the same procedure in the path integral formalism, it is possible to obtain the bubble loop contribution.

B.4 Ward Identity Application

Since the photon and Z boson are on-shell, the longitudinal polarization of the photon does not contribute to the amplitude, because there is only transverse modes,

$$\varepsilon_{\lambda_3}^* (p_3) \cdot p_3 = 0 \quad \forall \lambda = \pm. \quad (\text{B.41})$$

Consequently,

$$\varepsilon_{\mu}^{\lambda_2*} (p_2) \varepsilon_{\nu}^{\lambda_3*} (p_3) F_1 p_2^{\mu} p_3^{\nu} = 0 = \varepsilon_{\mu}^{\lambda_2*} (p_2) \varepsilon_{\nu}^{\lambda_3*} (p_3) F_3 p_3^{\mu} p_3^{\nu}, \quad (\text{B.42})$$

and the form factors F_1 and F_3 does not contribute to the amplitude. On the other hand, by means of the Ward-Takahashi identity, we note that

$$\begin{aligned} \varepsilon_{\mu}^{\lambda_2*} (p_2) p_{3\nu} \mathcal{R}^{\mu\nu} &= 0 = p_{3\nu} \varepsilon_{\mu}^{\lambda_2*} (p_2) [F_0 g^{\mu\nu} + F_2 p_2^{\mu} p_2^{\nu} + F_4 p_3^{\mu} p_2^{\nu}] \\ 0 &= \varepsilon_{\mu}^{\lambda_2*} (p_2) [F_0 p_3^{\mu} + F_2 (p_2 \cdot p_3) p_2^{\mu} + F_4 p_3^{\mu} (p_2 \cdot p_3)] \\ \therefore F_0 p_3^{\mu} + F_4 p_3^{\mu} (p_2 \cdot p_3) &= 0 \quad \wedge \quad F_2 p_2^{\mu} (p_2 \cdot p_3) = 0 \rightarrow F_2 = 0, \end{aligned} \quad (\text{B.43})$$

this approach allows us to consider only the contributing terms and necessary tensor structures for the amplitude, avoiding trivial simplifications in the computation.

B.5 Total Decay Width

In order to obtain a reduced expression of the decay width, we start from the case to $N = 2$ of the formula (2.28),

$$\begin{aligned} d\Gamma &= \frac{1}{2M} |\overline{\mathcal{M}}|^2 (2\pi)^4 \delta^{(4)} (P - p_1 - p_2) \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} \\ &= \frac{1}{2M} |\overline{\mathcal{M}}|^2 \delta^{(4)} (P - p_1 - p_2) \frac{d^3 p_1}{2E_1} \frac{d^3 p_2}{(2\pi)^2 2E_2} \\ &= \frac{1}{8\pi^2 M} |\overline{\mathcal{M}}|^2 \delta^{(4)} (P - p_1 - p_2) \frac{d^3 p_1 d^3 p_2}{4E_1 E_2}. \end{aligned} \quad (\text{B.44})$$

If we apply an integral over the momentums:

$$\begin{aligned} &= \frac{1}{8\pi^2 M} \int |\overline{\mathcal{M}}|^2 \delta^{(4)} (P - p_1 - p_2) \frac{d^3 p_1 d^3 p_2}{4E_1 E_2} \\ &= \frac{1}{8\pi^2 M} \int |\overline{\mathcal{M}}|^2 \delta^{(4)} (P^0 - E_1 - E_2) \delta^{(3)} (\vec{P} - \vec{p}_1 - \vec{p}_2) \frac{d^3 p_1 d^3 p_2}{4E_1 E_2}, \end{aligned} \quad (\text{B.45})$$

and, in the center-of-mass frame, $P = (M, \vec{0})$:

$$\begin{aligned} d\Gamma &= \frac{1}{8\pi^2 M} \int |\overline{\mathcal{M}}|^2 \delta (M - E_1 - E_2) \delta^{(3)} (-\vec{p}_1 - \vec{p}_2) \frac{d^3 p_1 d^3 p_2}{4E_1 E_2} \\ &= \frac{1}{8\pi^2 M} \int |\overline{\mathcal{M}}|^2 \delta (M - E_1 - E_2) \frac{(-d^3 p_1)}{4E_1 E_2}, \end{aligned} \quad (\text{B.46})$$

because of the Dirac's delta imposed $E_2 = \sqrt{m_2^2 + |\vec{p}_1|^2}$ and $E_1 = \sqrt{m_1^2 + |\vec{p}_1|^2}$,

$$\begin{aligned}
d\Gamma &= \frac{1}{8\pi^2 M} \int |\mathcal{M}|^2 \delta \left(M - \sqrt{m_1^2 + p_1^2} - \sqrt{m_2^2 + p_1^2} \right) \frac{(-d^3 p_1)}{4E_1 E_2} \\
&= \frac{1}{8\pi^2 M} \int |\mathcal{M}|^2 \delta \left(M - \sqrt{m_1^2 + p_1^2} - \sqrt{m_2^2 + p_1^2} \right) \frac{(-p_1^2 dp_1 \sin \theta_1 d\theta_1 d\phi_1)}{4E_1 E_2} \quad \because d^3 p_1 = p_1^2 dp_1 \sin \theta_1 d\theta_1 d\phi_1 \\
&= \frac{1}{8\pi^2 M} \int |\mathcal{M}|^2 \delta \left(M - \sqrt{m_1^2 + p_1^2} - \sqrt{m_2^2 + p_1^2} \right) \frac{p_1^2 dp_1 d\cos \theta_1 d\phi_1}{4E_1 E_2} \\
&= \frac{1}{8\pi^2 M} \int |\mathcal{M}|^2 \delta \left(M - \sqrt{m_1^2 + p_1^2} - \sqrt{m_2^2 + p_1^2} \right) \frac{p_1^2 dp_1}{4E_1 E_2} d\Omega.
\end{aligned} \tag{B.47}$$

In (B.47) we have an integral of the form $\int g(p_1) \delta(f(p_1)) dp_1$, therefore we apply the property of the Dirac's delta:

$$\delta(f(x)) = \left| \frac{df(x)}{dx} \right|_{x=x_0}^{-1} \delta(x - x_0), \tag{B.48}$$

where x_0 is the value for $\delta(f(x_0)) \neq 0 \Rightarrow f(x_0) = 0$. Therefore we can write the solution as follows:

$$\int |\mathcal{M}|^2 g(p_1) \delta(f(p_1)) dp_1 = |\mathcal{M}|^2 g(|\vec{p}_1|) \left| \frac{df(p_1)}{dp_1} \right|_{p_1=|\vec{p}_1|}^{-1}. \tag{B.49}$$

If we look the expression (B.47), we note that $\delta(f(p_1)) = \delta \left(M - \sqrt{m_1^2 + p_1^2} - \sqrt{m_2^2 + p_1^2} \right)$ and $g(p_1) = \frac{p_1^2}{4E_1 E_2}$, therefore we need to compute the following derivative:

$$\begin{aligned}
\left| \frac{df(p_1)}{dp_1} \right| &= \left| -\frac{1}{2} \frac{2p_1}{\sqrt{m_1^2 + p_1^2}} - \frac{1}{2} \frac{2p_1}{\sqrt{m_2^2 + p_1^2}} \right| \\
&= \frac{p_1}{\sqrt{m_1^2 + p_1^2}} + \frac{p_1}{\sqrt{m_2^2 + p_1^2}} = \frac{p_1}{E_1} + \frac{p_1}{E_2} \\
&= p_1 \left(\frac{E_1 E_2}{E_1 + E_2} \right).
\end{aligned} \tag{B.50}$$

Now, we compute the following expression with (B.50)

$$\begin{aligned}
g(|\vec{p}_1|) \left| \frac{df(p_1)}{dp_1} \right|_{p_1=|\vec{p}_1|}^{-1} &= \frac{|\vec{p}_1|^2}{4E_1 E_2} \frac{E_1 E_2}{|\vec{p}_1|(E_1 + E_2)} \\
&= \frac{|\vec{p}_1|}{4(E_1 + E_2)}.
\end{aligned} \tag{B.51}$$

Since $\delta(M - E_1 - E_2)$ imposes $M = E_1 + E_2$,

$$g(|\vec{p}_1|) \left| \frac{df(p_1)}{dp_1} \right|_{p_1=|\vec{p}_1|}^{-1} = \frac{|\vec{p}_1|}{4M}. \tag{B.52}$$

Therefore, we can replace (B.52) in (B.49):

$$\int |\overline{\mathcal{M}}|^2 \delta(M - E_1 - E_2) \frac{p_1^2 dp_1}{4E_1 E_2} = |\overline{\mathcal{M}}|^2 \frac{|\vec{p}_1|}{4M}, \quad (\text{B.53})$$

thereby, we have that the partial decay width is obtained by replacing (B.53) in (B.47):

$$\begin{aligned} d\Gamma &= \frac{1}{8\pi^2 M} \int |\overline{\mathcal{M}}|^2 \delta\left(M - \sqrt{m_1^2 + p_1^2} - \sqrt{m_2^2 + p_1^2}\right) \frac{p_1^2 dp_1}{4E_1 E_2} d\Omega \\ &= \frac{1}{8\pi^2 M} |\overline{\mathcal{M}}|^2 \frac{|\vec{p}_1|}{4M} d\Omega \\ &= \frac{1}{32\pi^2} |\overline{\mathcal{M}}|^2 \frac{|\vec{p}_1|}{M^2} d\Omega \end{aligned} \quad (\text{B.54})$$

In order to obtain $|\vec{p}_1|$, we write:

$$\begin{aligned} f(|\vec{p}_1|) &= 0 \\ M - \sqrt{m_1^2 + |\vec{p}_1|^2} - \sqrt{m_2^2 + |\vec{p}_1|^2} &= 0 \\ M^2 - m_1^2 - m_2^2 &= 2|\vec{p}_1|^2 + 2\sqrt{(m_1^2 + |\vec{p}_1|^2)(m_2^2 + |\vec{p}_1|^2)} \\ \left(\frac{M^2 - m_1^2 - m_2^2}{2} - |\vec{p}_1|^2\right)^2 &= m_1^2 m_2^2 + m_1^2 |\vec{p}_1|^2 + m_2^2 |\vec{p}_1|^2 + |\vec{p}_1|^4. \end{aligned} \quad (\text{B.55})$$

And we can obtain the expression for $|\vec{p}_1|$:

$$\begin{aligned} |\vec{p}_1|^2 &= \frac{1}{4M^2} (M^4 - 2M^2 m_1^2 - 2M^2 m_2^2 + m_1^2 + m_2^2 - 2m_1^2 m_2^2) \\ |\vec{p}_1| &= \frac{1}{2M} \sqrt{M^4 - 2(m_1^2 + m_2^2)M^2 + m_1^2 + m_2^2 - 2m_1^2 m_2^2} \\ |\vec{p}_1| &= \frac{1}{2M} \sqrt{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]} \end{aligned} \quad (\text{B.56})$$

Now, in the case of the process $H \rightarrow Z\gamma$, we have that

$$\Gamma(H \rightarrow Z\gamma) = \frac{1}{32\pi^2} \frac{|\overline{\mathcal{M}}|^2}{M_H^2} \frac{1}{2M_H} (M_H^2 - M_Z^2) (4\pi). \quad (\text{B.57})$$

The unpolarized squared module of the amplitude can be expanded as follows,

$$\begin{aligned} |\overline{\mathcal{M}}|^2 &= \sum_{\lambda_2, \lambda_3} |F_4|^2 \left| \varepsilon_{\lambda_2}^{\mu*}(p_2) \varepsilon_{\lambda_3}^{\nu*}(p_3) \left(-(p_2 \cdot p_3) g^{\mu\nu} + p_3^\mu p_2^\nu \right) \right|^2 \\ &= |F_4|^2 \sum_{\lambda_2, \lambda_3} \left| -(p_2 \cdot p_3) \left(\varepsilon_{\lambda_2}^* \cdot \varepsilon_{\lambda_3}^* \right) + \left(p_3 \cdot \varepsilon_{\lambda_2}^* \right) \left(p_2 \cdot \varepsilon_{\lambda_3}^* \right) \right|^2 \\ &= |F_4|^2 \sum_{\lambda_2, \lambda_3} \left[(p_2 \cdot p_3)^2 \left(\varepsilon_{\lambda_2}^* \cdot \varepsilon_{\lambda_3}^* \right) (\varepsilon_{\lambda_2} \cdot \varepsilon_{\lambda_3}) - (p_2 \cdot p_3) (\varepsilon_{\lambda_3} \cdot p_2) (\varepsilon_{\lambda_2} \cdot p_3) (\varepsilon_{\lambda_2}^* \cdot \varepsilon_{\lambda_3}^*) \right. \\ &\quad \left. - (p_2 \cdot p_3) (\varepsilon_{\lambda_3}^* \cdot p_2) (\varepsilon_{\lambda_2}^* \cdot p_3) (\varepsilon_{\lambda_2} \cdot \varepsilon_{\lambda_3}) + (\varepsilon_{\lambda_3}^* \cdot p_2) (\varepsilon_{\lambda_3} \cdot p_2) (\varepsilon_{\lambda_2}^* \cdot p_3) (\varepsilon_{\lambda_2} \cdot p_3) \right]. \end{aligned} \quad (\text{B.58})$$

With the completeness relation for the polarization vectors in massless and massive

cases, we can proceed as follows. We will illustrate this with an example, focusing on the first term in the expression above:

$$\begin{aligned}
(p_2 \cdot p_3)^2 \sum_{\lambda_2, \lambda_3} (\varepsilon_{\lambda_2}^* \cdot \varepsilon_{\lambda_3}^*) (\varepsilon_{\lambda_2} \cdot \varepsilon_{\lambda_3}) &= (p_2 \cdot p_3)^2 \sum_{\lambda_2, \lambda_3} g_{\mu\nu} g_{\alpha\beta} \varepsilon_{\lambda_2}^{\mu*} \varepsilon_{\lambda_3}^{\nu*} \varepsilon_{\lambda_2}^\alpha \varepsilon_{\lambda_3}^\beta \\
&= -(p_2 \cdot p_3)^2 \sum_{\lambda_2} g_{\mu\nu} g_{\alpha\beta} g^{\nu\beta} \varepsilon_{\lambda_2}^{\mu*} \varepsilon_{\lambda_2}^\alpha \\
&= -(p_2 \cdot p_3)^2 g_{\mu\nu} \sum_{\lambda_2} \varepsilon_{\lambda_2}^{\mu*} \varepsilon_{\lambda_2}^\nu \\
&= -(p_2 \cdot p_3)^2 g_{\mu\nu} \left(-g^{\mu\nu} + \frac{p_2^\mu p_2^\nu}{M_Z^2} \right) \\
&= -(p_2 \cdot p_3)^2 \left(-4 + \frac{p_2^2}{M_Z^2} \right) \\
&= 3(p_2 \cdot p_3)^2.
\end{aligned} \tag{B.59}$$

Thereby, the unpolarized squared module of the amplitude reads:

$$\begin{aligned}
|\overline{\mathcal{M}}|^2 &= |F_4|^2 \left[3(p_2 \cdot p_3)^2 - (p_2 \cdot p_3)^2 \right] \\
&= |F_4|^2 \frac{(M_H^2 - M_Z^2)^2}{2},
\end{aligned} \tag{B.60}$$

as a consequence of the expression above, the decay width is given by:

$$\begin{aligned}
\Gamma(H \rightarrow Z\gamma) &= \frac{1}{32\pi} \frac{1}{M_H^3} |F_4|^2 (M_H^2 - M_Z^2)^3 \\
&= \frac{M_H^3}{32\pi} \left(1 - \frac{M_Z^2}{M_H^2} \right)^3 |F_4|^2.
\end{aligned} \tag{B.61}$$

B.6 Tensor Reduction

The triangle loop contribution can be performed as follows,

$$\begin{aligned}
i\mathcal{M}^{(b)}(H \rightarrow \gamma Z) &= -2 \frac{e^3 M_W c_W}{s_W^2} \varepsilon_{\lambda_2}^{\mu*}(p_2) \varepsilon_{\lambda_3}^{\nu*}(p_3) \int \frac{d^4 \ell}{(2\pi)^4} \left(\frac{g_{\rho\sigma} g^{\rho\alpha} g^{\tau\varepsilon} g^{\lambda\sigma}}{D_0 D_1 D_2} \right) \mathcal{V}_{\mu\alpha\tau}(-p_2, \ell, -\ell_2) \mathcal{V}_{\nu\varepsilon\lambda}(-p_3, \ell_2, -\ell_3) \\
&= -2 \frac{e^3 M_W c_W}{s_W^2} \varepsilon_{\lambda_2}^{\mu*}(p_2) \varepsilon_{\lambda_3}^{\nu*}(p_3) \int \frac{d^4 \ell}{(2\pi)^4} \left(\frac{g^{\tau\varepsilon} g^{\lambda\alpha}}{D_0 D_1 D_2} \right) \mathcal{V}_{\mu\alpha\tau}(-p_2, \ell, -\ell_2) \mathcal{V}_{\nu\varepsilon\lambda}(-p_3, \ell_2, -\ell_3) \\
&= -2 \frac{e^3 M_W c_W}{s_W^2} \varepsilon_{\lambda_2}^{\mu*}(p_2) \varepsilon_{\lambda_3}^{\nu*}(p_3) \int \frac{d^4 \ell}{(2\pi)^4} \left(\frac{1}{D_0 D_1 D_2} \right) \mathcal{V}_\mu^{\lambda\varepsilon}(-p_2, \ell, -\ell_2) \mathcal{V}_{\nu\varepsilon\lambda}(-p_3, \ell_2, -\ell_3).
\end{aligned} \tag{B.62}$$

Now, in order to reduce the loop integral, we first expand the numerator and retain

only the contributing tensor structures to $p_3^\mu p_2^\nu$:

$$\begin{aligned}
i\mathcal{M}^{(b)} \rightarrow & \left(2 \frac{e^3 c_W M_W}{s_W^2} \right) \int \frac{d^d \ell}{(2\pi)^d} \frac{\varepsilon_{\lambda_2}^{\mu*}(p_2) \varepsilon_{\lambda_3}^{\nu*}(p_3)}{M_W^4 D_0 D_1 D_2} \\
& \times \left\{ \ell_\mu \ell_\nu \left[-4DM_W^4 - \ell^2 (3(\ell \cdot p_2) + \ell \cdot p_3 + 3M_W^2) - (\ell \cdot p_2) (-2(\ell \cdot p_3) - 5M_W^2 + M_Z^2) \right. \right. \\
& \quad \left. \left. + M_W^2 (\ell \cdot p_3) - M_Z^2 (\ell \cdot p_3) + 2(\ell \cdot p_2)^2 + \ell^4 + 6M_W^4 - 4M_W^2 (p_2 \cdot p_3) \right] \right. \\
& \quad \left. + \ell_\mu p_{2\nu} \left[4DM_W^4 + \ell^2 (3(\ell \cdot p_2) + \ell \cdot p_3 + 4M_W^2 + M_Z^2) - (\ell \cdot p_2) (2(\ell \cdot p_3) + 7M_W^2 + M_Z^2) \right. \right. \\
& \quad \left. \left. + 3M_W^2 (\ell \cdot p_3) - M_Z^2 (\ell \cdot p_3) - 2(\ell \cdot p_2)^2 - \ell^4 - 7M_W^4 + M_W^2 M_Z^2 \right] \right. \\
& \quad \left. + p_{3\mu} \ell_\nu \left[6M_W^2 (\ell \cdot p_2) - M_W^2 \ell^2 + M_W^4 - 3M_W^2 M_Z^2 \right] \right. \\
& \quad \left. + p_{3\mu} p_{2\nu} \left[2(\ell \cdot p_2) M_W^2 - 3\ell^2 M_W^2 - 5M_W^4 + M_W^2 M_Z^2 \right] \right\}. \tag{B.63}
\end{aligned}$$

The tensor-reduced bubble loop contribution reads:

$$\begin{aligned}
i\mathcal{M}^{(c)} \rightarrow & \left(\frac{e^3 c_W M_W}{s_W^2} \right) \int \frac{d^d \ell}{(2\pi)^d} \frac{\varepsilon_{\lambda_2}^{\mu*}(p_2) \varepsilon_{\lambda_3}^{\nu*}(p_3)}{M_W^4 D_0 D_2} \\
& \times \left\{ \ell_\mu \ell_\nu \left[2\ell \cdot p_2 + 2\ell \cdot p_3 - 2\ell^2 + 4M_W^2 \right] + \ell_\mu p_{2\nu} \left[-\ell \cdot p_2 - \ell \cdot p_3 + \ell^2 - 2M_W^2 \right] \right. \\
& \quad \left. + p_{3\mu} \ell_\nu \left[-\ell \cdot p_2 - \ell \cdot p_3 + \ell^2 - 2M_W^2 \right] + p_{3\mu} p_{2\nu} \left[2M_W^2 \right] \right\}. \tag{B.64}
\end{aligned}$$

In order to understand the general procedure of reducing tensor structures, we are going to reduce the coefficient of $p_3^\mu p_2^\nu$ in (B.63),

$$\begin{aligned}
& \frac{p_{3\mu} p_{2\nu}}{D_0 D_1 D_2} \left[2(\ell \cdot p_2) M_W^2 - 3\ell^2 M_W^2 - 5M_W^4 + M_W^2 M_Z^2 \right] \\
& = p_{3\mu} p_{2\nu} \left[-\frac{3\ell^2 M_W^2}{D_0 D_1 D_2} + \frac{2(\ell \cdot p_2) M_W^2}{D_0 D_1 D_2} - \frac{5M_W^4}{D_0 D_1 D_2} + \frac{M_W^2 M_Z^2}{D_0 D_1 D_2} \right], \tag{B.65}
\end{aligned}$$

performing the terms,

$$\begin{aligned}
-\frac{3\ell^2 M_W^2}{D_0 D_1 D_2} &= \frac{-3\ell^2 M_W^2 + 3M_W^4 - 3M_W^4}{D_0 D_1 D_2} \\
&= -\frac{3M_W^2 \overbrace{(\ell^2 - M_W^2)}^{D_0}}{D_0 D_1 D_2} - \frac{3M_W^4}{D_0 D_1 D_2} \\
&= -\frac{3M_W^2}{D_1 D_2} - \frac{3M_W^4}{D_0 D_1 D_2}, \tag{B.66}
\end{aligned}$$

$$\begin{aligned}
\frac{2(\ell \cdot p_2) M_W^2}{D_0 D_1 D_2} &= -\frac{(\ell - p_2)^2 M_W^2 - \ell^2 M_W^2 - p_2^2 M_W^2}{D_0 D_1 D_2} \\
&= -\frac{\overbrace{[(\ell - p_2)^2 - M_W^2]}^{D_1} M_W^2 + M_W^4 - M_W^2 (\ell^2 - M_W^2) - M_W^4 - M_Z^2 M_W^2}{D_0 D_1 D_2} \\
&= -\frac{M_W^2}{D_0 D_2} + \frac{M_W^2}{D_1 D_2} + \frac{M_Z^2 M_W^2}{D_0 D_1 D_2},
\end{aligned} \tag{B.67}$$

therefore, if we substitute (B.66) and (B.67) into (B.65),

$$\begin{aligned}
p_{3\mu} p_{2\nu} &\left[-\frac{3M_W^2}{D_1 D_2} - \frac{3M_W^4}{D_0 D_1 D_2} - \frac{M_W^2}{D_0 D_2} + \frac{M_W^2}{D_1 D_2} + \frac{M_Z^2 M_W^2}{D_0 D_1 D_2} - \frac{5M_W^4}{D_0 D_1 D_2} + \frac{M_W^2 M_Z^2}{D_0 D_1 D_2} \right] \\
&= p_{3\mu} p_{2\nu} \left[-\frac{8M_W^4 - 2M_Z^2 M_W^2}{D_0 D_1 D_2} - \frac{M_W^2}{D_0 D_2} - \frac{2M_W^2}{D_1 D_2} \right]
\end{aligned} \tag{B.68}$$

We obtained a tensor-reduced expression, meaning terms without loop momenta in the numerator. A similar procedure applies to the remaining tensor structures in the loop integral.

B.7 Passarino-Veltman Functions

In order to perform the loop integral, we have the tensor reduction of the tensor Passarino-Veltman functions appearing in (3.14). The Passarino-Veltmans in the amplitude are $A_{\mu\nu}^{(2,0)}$, $B_{\mu\nu}^{(01,02)}$, $C_{\mu\nu}^{(012)}$, $A_\mu^{(2,0)}$, $B_\mu^{(12,02,01)}$, $C_\mu^{(012)}$, $B_0^{(12,02)}$, $C_0^{(012)}$. The idea is to apply tensor reduction for the final tensor loop structures $\ell^\mu \ell^\nu$, $p_3^\mu \ell^\nu$, $\ell^\mu p_2^\nu$ and $p_3^\mu p_2^\nu$, make simplifications, and to remain with only the $p_3^\mu p_2^\nu$ terms, which yield to the reduced expression in (3.15). The tensor Passarino-Veltman functions are reduced as follows [53, 55].

We adopt the tensor reduction convention from [61], which aligns with the FeynCalc convention. Additionally, the reduction of tensor Passarino-Veltman functions is outlined in [33].

$$A_\mu^{(i)} = -A_0^{(i)} p_{(i)\mu}, \quad (\text{B.69})$$

$$B_\mu^{(0i)} = B_1^{(0i)} p_{(i)\mu}, \quad (\text{B.70})$$

$$B_{\mu\nu}^{(0i)} = B_{00}^{(0i)} g_{\mu\nu} + B_{11}^{(0i)} p_{(i)\mu} p_{(j)\mu}, \quad (\text{B.71})$$

$$B_\mu^{(12)} = \frac{B_0^{(12)}}{2} p_{3\mu} + B_0^{(12)} p_{2\mu}, \quad (\text{B.72})$$

$$B_{\mu\nu}^{(12)} = B_{00}^{(12)} g_{\mu\nu} + B_0^{(12)} p_{2\mu} p_{2\nu} + \frac{B_0^{(12)}}{3} p_{3\mu} p_{3\nu} + \frac{B_0^{(12)}}{2} (p_{2\mu} p_{3\nu} + p_{3\mu} p_{2\nu}), \quad (\text{B.73})$$

$$C_\mu^{(012)} = C_\mu = C_1 p_{2\mu} + C_2 p_{3\mu}, \quad (\text{B.74})$$

$$C_{\mu\nu}^{(012)} = C_{\mu\nu} = g_{\mu\nu} C_{00} + p_{2\mu} p_{2\nu} C_{11} + p_{3\mu} p_{3\nu} C_{22} + (p_{2\mu} p_{3\nu} + p_{3\mu} p_{2\nu}) C_{12}, \quad (\text{B.75})$$

and, since we are interested in contributions for $p_{3\mu} p_{2\nu}$, we need only to compute $B_{11}^{(12)}$, C_{12} , C_2 , $A_0^{(2)}$, $B_0^{(12)}$, C_0 , $B_1^{(02)}$. Moreover, there are tensor structures that will not contribute, $A_{\mu,\nu}^{(0)}$, $p_{2\nu} A_\mu^{(2)}$, $B_{\mu,\mu\nu}^{(01)}$. By tensor reduction, they imply a contribution to other tensor structures different from $p_{3\mu} p_{2\nu}$. Moreover, (B.69) is obtained as follows [62],

$$I = \int \frac{d^d \ell}{(2\pi)^d} \frac{1}{(k^2 + 2k \cdot p - M^2)^\alpha} = \frac{i\pi^{\frac{d}{2}}}{(2\pi)^d} \frac{(-1)^\alpha}{\Gamma(\alpha)} \frac{\Gamma\left(\alpha - \frac{d}{2}\right)}{(p^2 + M^2)^{\alpha - \frac{d}{2}}}, \quad (\text{B.76})$$

$$\begin{aligned} I_\mu &= \frac{-1}{2(\alpha-1)} \frac{\partial}{\partial p^\mu} \int \frac{d^d \ell}{(2\pi)^d} \frac{1}{(k^2 + 2k \cdot p - M^2)^{\alpha-1}} \\ &= \frac{-1}{2(\alpha-1)} \frac{\partial}{\partial p^\mu} \left[\frac{i\pi^{\frac{d}{2}}}{(2\pi)^d} \frac{(-1)^{\alpha-1}}{\Gamma(\alpha-1)} \frac{\Gamma\left(\alpha - \frac{d}{2} - 1\right)}{(p^2 + M^2)^{\alpha - \frac{d}{2} - 1}} \right] \\ &= -p^\mu \frac{i\pi^{\frac{d}{2}}}{(2\pi)^d} \frac{(-1)^\alpha}{\Gamma(\alpha)} \frac{\Gamma\left(\alpha - \frac{d}{2}\right)}{(p^2 + M^2)^{\alpha - \frac{d}{2}}} = -p_\mu I. \end{aligned} \quad (\text{B.77})$$

The contributing coefficients are written in terms of the irreducible integrals [33, 56]:

$$A_0^{(2)} = m^2 (\Delta_\varepsilon - \ln m^2 + 1), \quad (\text{B.78})$$

$$B_{11}^{(12)} = \frac{B_0^{(12)}}{3}, \quad (\text{B.79})$$

$$B_1^{(12)} = -\frac{B_0^{(12)}}{2}, \quad (\text{B.80})$$

$$B_0^{(12)} = \Delta_\varepsilon - \ln m^2, \quad (\text{B.81})$$

$$C_2 = \frac{(M_H^2 + M_Z^2) (B_0^{(02)} - B_0^{(12)}) - 2M_Z^2 (B_0^{(01)} - B_0^{(12)})}{(M_H^2 - M_Z^2)^2} - \frac{M_Z^2 C_0}{M_Z^2 - M_H^2} \quad (\text{B.82})$$

$$C_{12} = \frac{(M_Z^4 - M_H^4) B_0^{(01)}}{2(M_Z^2 - M_H^2)^3} + \frac{[M_Z^2 (5M_H^2 + M_Z^2) + M_H^4 - M_Z^4] B_0^{(02)}}{2(M_Z^2 - M_H^2)^3} \\ + \frac{(M_Z^2 + M_H^2) B_0^{(12)}}{2(M_Z^2 - M_H^2)^2} - \frac{M_W^2 (M_Z^2 + M_H^2) C_0}{(M_Z^2 - M_H^2)^2} + \frac{A_0^{(1)} - A_0^{(0)}}{(M_Z^2 - M_H^2)^2} - \frac{M_Z^2 + M_H^2}{2(M_Z^2 - M_H^2)^2}. \quad (\text{B.83})$$

With all these expressions, the form factor F_4 can be reduced to the expression (3.15), which involves only B_0 and C_0 functions. Consequently, their analytical expressions are required.

Now, consider the two-point irreducible scalar integral [53],

$$i\pi^2 B_0(p^2, M_W^2, M_W^2) = \mu^{4-d} \int \frac{d^d \ell}{(2\pi)^d} \frac{1}{D_0 D_1} \\ = \Delta_\varepsilon + B_0^{\text{fin}}(p^2; M_W^2, M_W^2). \quad (\text{B.84})$$

With the divergent part

$$16\pi^4 \Delta_\varepsilon = \frac{1}{\varepsilon} - \gamma_E + \ln 4\pi,$$

where γ_E is the Euler-Mascheroni constant, and the finite part of the scalar integral

$$16\pi^4 B_0^{\text{fin}}(p^2; m^2, m^2) = \ln\left(\frac{\mu^2}{m^2}\right) + 2 + \frac{\sqrt{p^2(p^2 - 4m^2)}}{p^2} \ln\left(\frac{\sqrt{p^4 - 4m^2 p^2} + 2m^2 - p^2}{2m^2}\right). \quad (\text{B.85})$$

In order to have a more compact expression, we are going to take a look on the argument of the logarithm

$$\begin{aligned}
\frac{\sqrt{p^4 - 4m^2 p^2} + 2m^2 - p^2}{2m^2} &= \frac{p^2 \sqrt{1 - \frac{4m^2}{p^2}} + 2m^2 - p^2}{2m^2} \left(\frac{\sqrt{1 - \frac{4m^2}{p^2}} + 1}{\sqrt{1 - \frac{4m^2}{p^2}} - 1} \right) \\
&= \frac{p^2 \left(1 - \frac{4m^2}{p^2} - 1 \right) + 2m^2 \left(\sqrt{1 - \frac{4m^2}{p^2}} + 1 \right)}{2m^2 \left(\sqrt{1 - \frac{4m^2}{p^2}} + 1 \right)} \\
&= \frac{-4m^2 + 2m^2 \left(\sqrt{1 - \frac{4m^2}{p^2}} + 1 \right)}{2m^2 \left(\sqrt{1 - \frac{4m^2}{p^2}} + 1 \right)} \\
&= \frac{-2 + \sqrt{1 - \frac{4m^2}{p^2}} + 1}{\left(\sqrt{1 - \frac{4m^2}{p^2}} + 1 \right)} = \frac{\sqrt{1 - \frac{4m^2}{p^2}} - 1}{\sqrt{1 - \frac{4m^2}{p^2}} + 1}.
\end{aligned} \tag{B.86}$$

Thereby,

$$\begin{aligned}
(16\pi^4) B_0^{\text{fin}}(p^2; m^2, m^2) &= \ln \left(\frac{\mu^2}{m^2} \right) + 2 - \sqrt{1 - \frac{4m^2}{p^2}} \ln \left(\frac{\sqrt{1 - \frac{4m^2}{p^2}} + 1}{\sqrt{1 - \frac{4m^2}{p^2}} - 1} \right) \\
&= \ln \left(\frac{\mu^2}{m^2} \right) + 2 - \sqrt{1 - \frac{4m^2}{p^2}} \ln \left(\frac{\sqrt{1 - \frac{4m^2}{p^2}} + 1}{\sqrt{1 - \frac{4m^2}{p^2}} - 1} \right).
\end{aligned} \tag{B.87}$$

Now, with the identity 8 in P.56 of [57],

$$\ln \left(\frac{z+1}{z-1} \right) = -2i \operatorname{arccot}(-iz),$$

then

$$-\sqrt{1 - \frac{4m^2}{p^2}} \ln \left(\frac{\sqrt{1 - \frac{4m^2}{p^2}} + 1}{\sqrt{1 - \frac{4m^2}{p^2}} - 1} \right) = 2i \sqrt{1 - \frac{4m^2}{p^2}} \operatorname{arccot} \left(-i \sqrt{1 - \frac{4m^2}{p^2}} \right).$$

Moreover, since in our amplitude, the values for p^2 and m^2 are respectively $M_{H,Z}^2$ and M_W^2 , we are dealing with the kinematics condition $\frac{4m^2}{p^2} > 1$, therefore,

$$\begin{aligned}
&= 2i^2 \sqrt{\frac{4m^2}{p^2} - 1} \operatorname{arccot} \left(-i^2 \sqrt{\frac{4m^2}{p^2} - 1} \right) \\
&= -2 \sqrt{\frac{4m^2}{p^2} - 1} \operatorname{arccot} \left(\sqrt{\frac{4m^2}{p^2} - 1} \right),
\end{aligned}$$

and we use the identity 10 in P.57 in [57],

$$\operatorname{arccot} x = \arcsin \left(\frac{1}{\sqrt{1+x^2}} \right), \quad \forall x > 0 \tag{B.88}$$

$$\therefore -2\sqrt{\frac{4m^2}{p^2}} - 1 \operatorname{arccot}\left(\sqrt{\frac{4m^2}{p^2}} - 1\right) = -2\sqrt{\frac{4m^2}{p^2}} - 1 \arcsin\left(\frac{1}{\sqrt{\frac{4m^2}{p^2}}}\right), \quad \forall \frac{4m^2}{p^2} > 0,$$

thereby, we write the finite part of the two-point scalar integral as follows

$$16\pi^4 B_0^{\text{fin}}(p^2; m^2, m^2) = \ln\left(\frac{\mu^2}{m^2}\right) + 2 - 2\sqrt{\frac{4m^2}{p^2}} - 1 \arcsin\left(\frac{1}{\sqrt{\frac{4m^2}{p^2}}}\right). \quad (\text{B.89})$$

From the last procedure, we can conclude the following relation

$$\ln\left(\frac{\sqrt{p^4 - 4m^2 p^2} + 2m^2 - p^2}{2m^2}\right) = 2i \arcsin\left(\frac{1}{\sqrt{\frac{4m^2}{p^2}}}\right), \quad (\text{B.90})$$

Now, for the three-point scalar integral,

$$i\pi^2 C_0(p_1^2, p_2^2, (p_1 + p_2)^2; m_1, m_2, m_3) = \mu^{4-d} \int \frac{d^d \ell}{(2\pi)^d} \frac{1}{D_0 D_1 D_2}. \quad (\text{B.91})$$

For our case, we have the following expression [55]:

$$\begin{aligned} i\pi^2 C_0(0, M_H^2, M_Z^2; M_W^2, M_W^2, M_W^2) = \\ \frac{\log^2\left(\frac{\sqrt{M_H^4 - 4M_H^2 M_W^2} - M_H^2 + 2M_W^2}{2M_W^2}\right)}{32\pi^4 (M_H^2 - M_Z^2)} - \frac{\log^2\left(\frac{\sqrt{M_Z^4 - 4M_Z^2 M_W^2} - M_Z^2 + 2M_W^2}{2M_W^2}\right)}{32\pi^4 (M_H^2 - M_Z^2)}, \quad \forall \frac{4M_W^2}{M_{Z,H}^2} > 1, \end{aligned} \quad (\text{B.92})$$

and by means of the relation (B.90), we obtain:

$$i\pi^2 C_0(0, M_H^2, M_Z^2; M_W^2, M_W^2, M_W^2) = -\frac{\arcsin^2\left(\frac{1}{\sqrt{\frac{4M_W^2}{M_H^2}}}\right)}{8\pi^4 (M_H^2 - M_Z^2)} + \frac{\arcsin^2\left(\frac{1}{\sqrt{\frac{4M_W^2}{M_Z^2}}}\right)}{8\pi^4 (M_H^2 - M_Z^2)}. \quad (\text{B.93})$$